Public and Private Liquidity Providers

Arnoldo López-Marmolejo\textsuperscript{a} and Fabrizio López-Gallo Dey\textsuperscript{b,*}

\textsuperscript{a}BBVA Research
\textsuperscript{b}Banco de México, DGASF

May 31, 2010

Abstract
The goal of this paper is to explore the benefits of having a private liquidity provider and the conditions under which this lender provides liquidity, when a public liquidity provider is also present. The model proposed incorporates an endogenous interbank lending market so that the decision of a bank to seek liquidity in the interbank market or to turn to the private or the public lender is also endogenous. This framework permits the derivation of conclusions on the size of the private lender, interbank lending conditions and optimal policy for liquidity provision.

1 Introduction

A key responsibility of central banks is provision of liquidity to banks in periods of financial stress. However, there are privately-owned companies that also have that objective. In the United States, the Congress created the Federal Home Loan Banks (FHLBanks) as a system of government-sponsored enterprises, federally chartered but privately capitalized and independently managed, and gave them the mission, among other responsibilities, to serve as a reliable source of liquidity for their membership. The FHLBanks pursued their mission and provided liquidity to their member banks during the crisis. They had a very important role in liquidity provision after the bankruptcy of Lehman Brothers as other sources of liquidity such as the financial markets and the interbank market restrained. Their loans reached its maximum in the third quarter of 2008 (see Figure 1).

\textsuperscript{*}We would like to thank Jorge Sicilia, Ociel Hernández and seminar participants at the Bank of Mexico. E-mail: arnoldo.lopez@bbva.bancomer.com or flopezd@banxico.org.mx
Similar institutions exist also in other countries. For example, in Germany, the banks associations created the Liquiditäts-Konsortialbank or Liko bank to provide liquidity to its members (70% owned by individual members of the banking system and 30% by the Bundesbank). The purpose of this paper is to analyze the social benefits of the existence of these institutions and their interaction with the public lender of last resort.

Although the literature on the lender of last resort (LOLR) is extensive (Freixas et al., 2000 for a recent survey), to our knowledge, papers that analyze the interaction between private and public liquidity providers are scarce. However, some efforts have been made. For example, Herrala (2001) investigates whether voluntary schemes such as public lender of last resort, a mutual clearing house, and a profit-maximizing private LOLR are, in terms of social utility, as good as a compulsory LOLR scheme. Our paper, in contrast, permits the interaction between a bank that requires liquidity and the different potential providers. In order to analyze this interaction we construct a model where the decision of a bank to seek liquidity in the interbank market or to turn to a private or a public lender is endogenous. This framework allows us to derive conclusions on the size of the private lender, interbank lending conditions and optimal policy for liquidity provision. Research about optimal LOLR institutional design has been done (see Repullo, 2000). However, it has considered only public sector institutions: the central bank and the deposit insurance corporation.

The results derived from this paper are the following. First, the private
liquidity provider supplies resources to banks only when the amount lent is high enough. This could explain why the Federal Home Loan Banks hold total assets of around 6 per cent of US GDP. This result suggests a relevant corollary: a public liquidity provider is necessary for small liquidity provisions. Second, the less correlated the shocks across banks are, the more difficult interbank lending is, which makes a lender of last resort institution even more important. When the shocks are correlated a bank is more willing to lend to other bank as it can obtain some profit even if a bad shock hits both banks as this model permits the lending bank to keep some remnant from the project financed. This remnant can be seen as a collateral that may cover a share of the loan. Third, in the case that the amount of liquidity required is high and there is a stigma or reputational cost for a bank that obtains liquidity from the public lender, the existence of a private liquidity provider is welfare-improving. Notwithstanding, a public liquidity provider should exist as it improves welfare when the liquidity needs and the reputation costs are small. This result supports the need for a public lender of last resort.

The paper is organized as follows. Section 2 presents the model and social welfare analysis for different liquidity provision arrangements. Section 3 provides some concluding remarks.

2 Model

This model assumes an economy that is formed by two banks that act as local monopolies and face a continuum of consumers of size one. There is a public deposit insurance corporation that may also act as a liquidity provider and a private liquidity provider.

The economy has three periods. At period $t = 0$ each bank raises one unit of deposits. For simplicity we assume that consumers are unable to store consumption goods, and hence the bank provides a valuable service since consumers prefer to consume at $t = 2$. At $t = 1$ consumers may receive a shock that forces them to withdraw their deposits. The fraction of deposits withdrawn early is a random variable $0 \leq v \leq 1$ with distribution function $F(v)$ and density $f(v) < z$.

The banks are endowed with a constant returns to scale technology that yields a different return depending on the state of the world. In state high, $h$, it yields $R_h$ whereas in state low, $l$, it yields $R_l$. Let $R_l < 1 < R_h$. The probability of state $h$ is $p$ and of state $l$ is the complementary probability. We also have that $\bar{R} = pR_h + (1 - p)R_l > 1$, and therefore the project makes sense since it has positive expected value. The technology requires two periods to mature. Thus, it delivers the returns in period $t = 2$. Early liquidation is possible, but with a cost. Let $L \leq 1$ be the total resources obtained after liquidation. The difference $1 - L$ represents the cost of early liquidation.

For explanatory purposes consider first no liquidity from outside sources. Hence we have that at $t = 0$ consumers deposit 1 at the bank, at $t = 1$ the value of $v$ is realized and the bank has to liquidate part of the project, if it is enough to pay $v$ the bank continues to next period, otherwise the bank fails and
the depositors get the liquidation value \( L \). Therefore, we have that the bank is able to continue for the next period if \( v < \hat{v} \), with \( \hat{v} = L \). If the bank survives in \( t = 2 \) the state of the world unfolds and the return is known. If the return of the project is enough to pay depositors, the rest is for the bank, if not, \( 1 - v \) depositorys get whatever is left. In state \( l \), patient depositors are not fully paid. Figure 2 summarizes the basic timing of events.

For explanatory purposes, consider first the problem when there is only a bank. This is, there is no interbank lending market. We will expand the analysis to incorporate the role of the interbank lending market in relation to other potential sources of liquidity in the next sub-section.

Let \( \alpha \) be the proportion of deposits required to be liquidated in order to pay the withdrawal at \( t = 1 \). So we have that \( \alpha L \) must equal \( v \), so \( \alpha = \frac{v}{L} \geq v \). The expected profits of bank 1 in the non-run equilibrium are:

\[
E(\pi_{B_1}) = \int_{0}^{v_1^*} p \left[ \left( 1 - \frac{v_1}{L_1} \right) R_h - (1 - v_1) \right] f(v_1)dv_1
\]

where the subscript 1 refers to the bank and \( v_1^* = \frac{(R_h - 1) L_1}{R_h - L_1} \). The value \( v_1^* \) represents the benchmark of \( v_1 \) that permits the bank to pay the patient consumer. For \( v_1 > v_1^* \) the return of the project is not enough to pay the patient depositors. Note that \( v_1^* < \hat{v}_1 = L_1 \). Also note that in state \( l \) returns are never enough for paying patient depositors. The bank does not have negative profits, and hence does not absorb losses to pay patient consumers, due to limited liability.

The deposit insurance institution (\( DI \)) could discard the run equilibria. The expected losses of the deposit insurance fund would be:

\[
E(\pi_{DI} \mid DI \text{ insure}, E(\pi_{B_1})) = \int_{0}^{v_1^*} (1 - p) \left[ (1 - \frac{v_1}{L_1}) R_l - (1 - v_1) \right] f(v_1)dv_1 + \int_{v_1^*}^{v_1} (1 - \frac{v_1}{L_4}) R - (1 - v_1) f(v_1)dv_1 + \int_{v_1}^{1} (L_1 - v_1) f(v_1)dv_1(1)
\]
Notice that where there is a DI the consumers lose nothing and they do not have any incentive to withdraw if they do not receive a shock.

Additionally, the deposit insurance may lend $v_1$ to the bank in order to avoid inefficient liquidation of the project. Assuming a zero interest rate for its lending, the expected loss of DI when it lends to the bank is

$$E(\pi_{DI} | DI \text{ lend}, E(\pi_{B1})) = (1 - p)(R_l - 1).$$ (2)

The expected profits of bank 1 when it obtains liquidity from DI are

$$E(\pi_{B1}) = p(R_h - 1) - c,$$ (3)

where $c$ is a non-pecuniary cost that has to be paid in case a bank receives a loan from DI. This cost can be seen as a reputation or stigma cost. Jenkinson, N. (2009) considers that this stigma implies that some central bank liquidity facilities are ineffective.¹

Define $\tau = p(R_h - 1)$, if $c = \tau$ bank 1 finds optimal to borrow from DI only if $v_1 > v^*_1$ and there is utility from finishing the project. Now for every $c < \tau$ we have a $v^*_1 = \frac{cL}{p(R_h - L)}$ such that bank 1 borrows only if $v_1 > v^*_1$. The parameter $v^*_1$ is the minimum value of $v_1$ that permits the bank to make more profits by obtaining credit from the DI than by liquidating the project. This implies that bank 1 asks for a loan only if the size of the withdrawal shock is high enough, otherwise it prefers to under-invest, increasing the cost of the DI in the l state:²

$$E(\pi_{DI} | DI \text{ lend, } E(\pi_{B1})) = \int_0^{v^*_1} (1 - p) \left[ (1 - \frac{v_1}{L_1})R_l - (1 - v_1) \right] f(v_1)dv_1 +$$

$$+ \int_{v^*_1}^1 (1 - p)(R_l - 1)f(v_1)dv_1$$

$$= (1 - p)(R_l - 1) - \int_0^{v^*_1} \frac{(1 - p)v_1}{L_1}R_ldv_1. $$

Allow a private liquidity provider (PL) to lend $v_1$ to the bank. The expected profit function of the PL is:

$$E(\pi_{PL}) = p(R_h - (1 - v_1)) + (1 - p)[R_l - (1 - v_1)]^+ - v_1$$ (4)

The factor $(1 - v_1)$ represents the share of deposits that have to be given back to the depositors. This result assumes that the lender expropriates all profits from the bank. This assumption does not seem to be highly unrealistic, as the liquidity provision could be seen as a monopolistic activity. In addition,

¹Nigel Jenkinson was Executive Director of the Financial Stability Area in the Bank of England.

²Notice that the public lender always finds it optimal to supply liquidity because in the case of bankruptcy it would bear the cost of the inefficiency created by the early liquidation.
this assumption only alters the distribution of profits among agents. As we are analyzing the effect on social welfare of the introduction of a private liquidity provider, a different distribution of profits among agents does not alter the results.

Notice that there exists a small \( v_1 \) that makes \( E(\pi_{PL}) < 0 \). This happens for \( v_1 < \tilde{v}_1 \), where \( \tilde{v}_1 = 1 - R_f \) (see proof in the annex). This implies that the \( PL \) provides liquidity only when the amount to be lent (withdrawal shock) is high enough (higher than threshold \( \tilde{v}_1 \)). The intuition behind is that the expected benefits to \( PL \) are enough to pay its investment only when the patient depositors, which are paid first, are few, and therefore it can expropriate enough resources. As a bank failure has detrimental effects on the functioning of the economy (see Goodhart, 1995), the previous result implies that a public lender should exist and act when the \( PL \) is not willing to do so (assuming the absence of an interbank market). Bank 1 borrows from the \( PL \) only if \( v_1 > v_1^* \) and \( c > \bar{c} \).

Figure 3 shows under which conditions \( PL \) and \( DI \) provide liquidity and bank 1 chooses to liquidate its project.\(^3\) When the cost for the bank of receiving a loan from \( DI \) is high enough (i.e. \( c > \bar{c} \)), bank 1 chooses to liquidate the project when the early deposits withdrawal is small enough to obtain profits (\( v_1 < L \)). When the early withdrawal does not permit bank 1 to obtain profits, it would be indifferent between obtaining liquidity from \( PL \) or liquidating the project. As bank 1 can survive by borrowing from \( PL \) we assume that it prefers to borrow and survive than to liquidate the project and abandon the market. In addition, obtaining liquidity from \( PL \) is a Pareto improvement as bank 1 remains as before, \( PL \) obtains profits and patient consumers get back their deposits. The dark area indicates when the \( PL \) provides liquidity. The results are supported by Bech et al. (2007) who find that the use of FHLBank advances by financial institutions in contrast to borrowing from the discount window, is the result of a lack of stigma, among other factors.

Bank 1 chooses to obtain a loan from \( DI \) rather than to liquidate the project when the deposits withdrawal is high enough to obtain profits after facing the cost of receiving the loan (i.e. \( v_1 > v_1^* \)). The area where this occurs is indicated by the letters \( DI \). For values \( v_1 > L \), obtaining liquidity from \( DI \) is a Pareto improvement as patient consumers obtain all their deposits, the whole project matures, and therefore the output is higher.

\(^3\)For illustration purposes in this figure we assume that \( L_1 > (1 - R_f) \).
2.1 Interbank Lending Market

In order to expand the analysis, we now consider bank 2, $B_2$, and the role of the interbank lending market in relation to other potential liquidity providers. For simplicity, a scale problem is assumed so that banks can receive no more than 1 in deposits. In this model, assume that bank 2 is efficient, meaning it can fully liquidate its assets at time 2 without cost, $L_2 = 1$. The expected profits of bank 2 at $t = 1$ are:

$$E(B_2) = p(R_h - 1)(1 - v_2)$$

Bank 2 asks for a loan to the $DI$ if and only if $c < v_2 p(R_h - 1)$. There exists a $v_2^* = \frac{c}{p(R_h - 1)}$ such that $B_2$ finds it attractive to borrow from the $DI$. In such a case $E(\pi_2) = p(R_h - 1) - c$. But notice that $B_2$ only borrows in order to obtain higher profits, so we can assume that the $DI$ refuses to lend in that case. In the case of the private liquidity provision, $B_2$ never finds it optimal to borrow from the $PL$ because it would lose all of its profits. Notice also that $B_2$ cannot borrow from the $DI$ and use these resources to lend to $B_1$ in order to obtain profits using public funds.

The expected losses for the insurance fund from this bank are:

$$E(\pi_{DI} \mid_{DI \text{ insure}}, E(\pi_{B_2})) = (1-p) [(1 - v_2)R_l - (1 - v_2)] = (1-p)(R_l-1)(1-v_2)$$

Given that bank 2 is efficient it could lend to bank 1 at $t = 1$. Ex-ante expectations about the size of withdrawal shocks or loans are not important given that no decision has to be made at $t = 0$. The decision to lend and borrow
will be made once $v_1$ and $v_2$ are known. The timing of events when bank 1 has access to liquidity sources is described in Figure 4.

The possibility of an interbank market enriches the decision space with four possible outcomes at $t = 2$. Let $y$ be the amount bank 2 lends to bank 1 and $R$ be the interest rate on the loan, $R = 1 + r$. For simplicity, in our framework the surplus bank providing liquidity does not have market power in the interbank market, and thus, it does not strategically under-provide lending as in Acharya, Gromb and Yorulmazer (2008).

The decision for bank 1 on whether to take funds from bank 2 depends on the values of $v_1$ and $c$, given that both the public and the private liquidity provider exist in this world. Hence, the decision for bank 1 could be described as:

Borrow if $E(\pi_{B_1} | y) \geq \begin{cases} 0 & \text{if } c > \Bar{c} \text{ and } v_1 > v_1^* \\ p(R_h - 1 - c) & \text{if } c < \Bar{c} \text{ and } v_1^* \geq v_1 > v_1^c \\ p \left(1 - \frac{r}{1 + R_h} \right) R_h - (1 - v_1) & \text{if } c < \Bar{c} \text{ and } v_1 \leq v_1^c. \end{cases}$

In the last two cases bank 1 borrows from bank 2 if $R \leq \frac{R_h}{2}$. This is when the price bank 1 has to pay to bank 2 for the liquidity is smaller than the potential benefit of the project adjusted by the liquidation efficiency. In the first case of the inequality, the set of values where bank 1 borrows from bank 2 exist but are hard to achieve given that $v_1$ is large, bank 2 can only lend from its own funds and $R < \frac{R_h}{2}$. Although it is possible that the interbank market exists in such an environment, the probability is small. Notice that the decision of bank 1 does not depend on the state of the world that affects bank 2. However, the way events unfold in $t = 2$ is relevant for bank 2.

Let the pair $(a, b)$ represent the state of nature at $t = 2$, the first coordinate corresponds to the state bank 1 faces and the second to the state of bank 2.

Figure 4. Timing of events.
Profits for bank 2 when it lends to bank 1 are:

$$
\pi_{B_2} = \begin{cases} 
(1 - v_2)R_h - (1 - v_2) + y(R - R_h) & \text{if } (h, h) \\
(1 - v_2)R_h - (1 - v_2) + t - yR_h & \text{if } (l, h) \\
(1 - v_2)R_l - (1 - v_2) + y(R - R_l) & \text{if } (h, l) \\
(1 - v_2)R_l - (1 - v_2) + t - yR_l & \text{if } (l, l)
\end{cases}
$$  \hspace{1cm} (7)

with \( t = \left(1 - \frac{v_1 - y}{R_l - (1 - v_1)}\right) \) being the remnant from bank 1 in state \( l \). In order to calculate the \( E(\pi_{B_2}) \) the probability of occurrence of each state for each bank is needed. Since we have only two states, \((h, l)\), a Bernoulli distribution is appropriate. It is assumed that the states \( h \) and \( l \) for each bank are correlated with parameter \( \theta \). The probability matrix is then given by:

$$
\begin{bmatrix}
P(h, h) & P(h, l) \\
P(l, h) & P(l, l)
\end{bmatrix} = \begin{bmatrix}
p^2 + \theta p(1 - p) & p(1 - p)(1 - \theta) \\
p(1 - p)(1 - \theta) & (1 - p)^2 + \theta p(1 - p)
\end{bmatrix}
$$  \hspace{1cm} (8)

The expected profits are subject to limit liability, so depending on the size of \( R \) and \( y \) the expected profit in a state may be zero. The expected profits function for bank 2 is:

$$
E(\pi_{B_2} | y) = \begin{cases} 
p [(1 - v_2)(R_h - 1) - yR_h] + dyR + et & \text{if } y < \tilde{y} \\
p [(1 - v_2)(R_h - 1) + y(R - R_h)] + e [t + (1 - v_2)(R_l - 1) - yR_l] & \text{if } y \in [\tilde{y}, \hat{y}] \\
d [(1 - v_2)(R_h - 1) - yR_h] + e [(1 - v_2)(R_l - 1) - yR_l] + pyR & \text{if } y > \hat{y}
\end{cases}
$$  \hspace{1cm} (9)

Where \( \tilde{y} = (1 - v_2) \left(1 - \frac{1}{R_h}\right) \) and \( \hat{y} = (1 - v_2) \left(\frac{1 - R_l}{R - R_h}\right) \) represent the thresholds of \( y \) that permit bank 2 to make positive profits and therefore be willing to give a loan. Let \( d = p^2 + \theta p(1 - p) \) and \( e = p(1 - p)(1 - \theta) \).

When \( R \) is large enough the \( E(\pi_{B_2} | y) \) is lowest when \( y < \tilde{y} \). The expected profits of bank 1 when it receives a loan from bank 2 are:

$$
E(\pi_{B_1} | y) = \begin{cases} 
p [(1 - v_1 - y)R_h - (1 - v_1) - yR_h] & \text{if } (h, h) \text{ or } (h, l) \\
0 & \text{otherwise.}
\end{cases}
$$  \hspace{1cm} (10)

The probability matrix allows us to obtain a scalar of the previous function.

For bank 2 it is optimal to give a loan if \( E(\pi_{B_2} | y < \tilde{y}) > E(\pi_{B_2} | y = 0) \). This occurs when

$$
R > \frac{R_h}{p + \theta(1 - p)}.
$$

When \( \theta \to -1 \) it is less likely that the inequality holds (as long as \( p > \frac{1}{2} \)). That is, the less correlated the shocks are, the more difficult it is for bank 2 to give a loan. Although this is not an obvious finding, the intuition that lies behind is that when the shocks are correlated, bank 2 is more willing to lend to
bank 1 in the high state as it recovers its loan and makes profits, and in the low
state instead of having only losses it keeps the return from the loan to bank 1.

That lies behind is the following. There are two states, high and low. When
the shocks are correlated, in the high state bank 2 lends to bank 1 as it recovers
its loan and makes profits. In the low state, bank 2 instead of having only losses
it keeps the return from the loan to bank 1.

When bank 2 does not give a loan to bank 1 (e.g. \( R > \frac{R_h}{p + \sigma (1 - p)} \) does not
hold) bank 1 has to borrow from the private lender or the deposit insurance
institutions.

Bank 1 prefers to obtain a loan from bank 2 rather than to liquidate when
\( E(\pi_{B_1} | y) > E(\pi_{B_1} | \text{Liquidate}) \). This occurs as long as

\[
R \leq \frac{R_h}{L}.
\]

This is, the payoff for bank 1 in the high state, adjusted for its liquidation
efficiency, is higher than the cost of a loan from bank 2.

Bank 1 prefers to obtain a loan from bank 2 rather than from DI when
\( E(\pi_{B_1} | y) > E(\pi_{B_1} | DI \text{ lend}) \). This is when

\[
v_1 \leq v_1^{DI} = \frac{y(R_h - LR)}{R_h - L} + \frac{L}{(R_h - L)} \frac{c}{p}.
\]

When the loan required is high enough, \( v_1 > v_1^{DI} \), bank 1 prefers not to
borrow from bank 2, but rather to borrow from DI as the return of the project
is not enough to pay bank 2 \( yR \) and obtain profits higher than those obtained
by borrowing from DI.

Bank 1 prefers to obtain a loan from bank 2 rather than from PL when
\( E(\pi_{B_1} | y) > E(\pi_{B_1} | PL) \). This is when

\[
v_1 \leq v_1^{PL} = \frac{y(R_h - LR)}{R_h - L} + \frac{L(R_h - 1)}{(R_h - L)}.
\]

When the loan required by bank 1 from bank 2 is small enough, \( v_1 \leq v_1^{PL} \),
the benefits of the project are enough to pay back bank 2 and make positive
profits.

The next figure shows the areas where each agent provides liquidity to bank
1.
2.2 Social Welfare Analysis

The social welfare is defined as the sum of benefits/costs of bank 1, bank 2, the private liquidity provider and the deposit insurance institution, and is calculated for each one of the liquidity provision outcomes. For illustration purposes we assume a uniform density function, $f(v) = 1$, $0 < v < 1$.

First, consider that bank 2 does not provide liquidity to bank 1. (e.g. $R > \frac{R_h}{p+(1-p)}$ does not hold). In this case bank 1 can obtain liquidity from the PL, the DI or can decide to liquidate its project. The social welfare associated to each one of these three outcomes is the following:

Case 1. Only PL provides liquidity (i.e. $c > \bar{c}$ and $v_1 > v_1^+$).

$$W(v) = p(R_h - 1)(1 - v_2) + [p(R_h - (1 - v_1)) + (1 - p)[R_l - (1 - v_1)]^+] - v_1 - (1 - p)(R_l - 1)(1 - v_2) = \frac{R_h}{1} - 2 - v_2. \quad (11)$$

Let $a = (\bar{R} - 1)(2 - v_2)$. This represents the total welfare of the economy.

Case 2. Only DI provides liquidity (i.e. $c < \bar{c}$ and $v_1 > v_1^+$).

$$W(v) = p(R_h - 1) - c + p(R_h - 1)(1 - v_2) + (1 - p)(R_l - 1) + (1 - p)(R_l - 1)(1 - v_2) = (\bar{R} - 1)(2 - v_2) - c = a - c. \quad (12)$$
Case 3. Neither PL nor DI provide liquidity.

\[ W(v) \mid \text{No Liquidity} = a + \overline{R} \left( \frac{L}{2} - 1 \right) + \frac{1}{2} \]  \hspace{1cm} (13)

The scalar \( \overline{R} \left( \frac{L}{2} - 1 \right) + \frac{1}{2} \) represents the cost of bank 1’s project not being able to mature.

Now, consider the social welfare when there is interbank liquidity.

Case 4. Bank 2 provides liquidity to bank 1.

\[ W(v) \mid \text{Bank 2 provides liquidity} = a - \left[ y(\overline{R} - 1 + R(\theta - 1)(p - p^2)) - et \right] \]  \hspace{1cm} (14)

The expression \( - [y(\overline{R} - 1 + R(\theta - 1)(p - p^2)) - et] \) characterizes the cost to not permit to the share \( y \) of the project of bank 2 to mature plus an expected remnant from bank 1 in state \( l \), in case it exists. The liquidation of a share of the project implies that the output of the economy is below its potential.

Figure 6 shows the pattern of social welfare in each case.

Figure 6. Social welfare.

This figure shows four implications of the model.

1. When \( v_1 > 1 - R_l \), it is better that the PL provides liquidity rather than the DI. This occurs due to the reputational cost of obtaining liquidity from the DI.

2. Notice that the lower \( R_l \) is, the lower are the values of \( v \) for which the PL provides liquidity, as it would not obtain enough profits in the low state.
3. In terms of social welfare, it is better that the $DI$ provides liquidity than the bank 2 does it and than to have no liquidity at all, as long as the non-pecuniary cost, $c$, is smaller than the cost of the project not being able to mature.

4. When $v_1 < 1 - R_l$ there is a $v_1$, $v_1 > v_1^c$, where $DI$ provides liquidity and $PL$ does not. This occurs when

$$c < \frac{p(R_h - L)(1 - R_l)}{L}$$

as bank 1 is willing to obtain liquidity from $DI$ when the reputation cost, $c$, is low enough. This implies that for small shocks, the presence of $DI$ is welfare-improving.

The difference between $W(v) \mid_{No\ Liquidity}$ and $W(v) \mid_{Bank\ 2\ provides\ liquidity}$ depends on the share $y$ of the project of bank 2 that was not able to mature, its return, the cost of the loan from bank 2 and the resources obtained after bank 1 liquidates its project, among others. For illustration purposes, in this graph has been assumed that the cost of bank 1's project is larger than the cost of the share $y$ of the project of bank 2 when they are not being able to mature. This does not seem unrealistic considering that bank 2 is more efficient in liquidating its project.

3 Conclusions

The conclusions of this paper can be summarized as follows:

1. The $PL$ provides liquidity only when the amount lent (withdrawal shock) is high enough, so private liquidity providers are expected to be large. This may explain why the FHLBanks hold total assets that amount to 6 per cent of US GDP.

2. The less correlated the shocks across banks are, the harder it is to rely on interbank lending.

3. When the shock is large enough and the provision of liquidity by $DI$ imposes reputation costs, it is better for $PL$ to provide liquidity than for $DI$ to do it.

4. $DI$ should exist as its presence is welfare-improving when the withdrawal shock and the reputation costs are small.

5. In terms of social welfare, the interbank market is the worst of the liquidity provision options as a bank decides to liquidate a share of its project to provide liquidity to other bank. In contrast, when $PL$ or $DI$ provides liquidity no project has to be liquidated.

Annex

Proof. For $PL$, there exists a small $v_1$ that makes $E(\pi_{PL}) < 0$. 

13
The expected profits function of $PL$ is given by $E(\pi_{PL}) = p(R_h - (1 - v_1)) + (1 - p)[R_l - (1 - v_1)|^+ - v_1$. We can identify two cases, where $|R_l - (1 - v_1)|$ is positive and zero, as banks have limited liability.

Case 1. $|R_l - (1 - v_1)| > 0$ or what is the same, $v_1 > \bar{v}_1$, where $\bar{v}_1 = 1 - R_l$. In this case the expected profits function becomes $E(\pi_{PL}) = \bar{R} - 1 > 0$.

Case 2. $|R_l - (1 - v_1)| = 0$ or what is the same, $v_1 < \bar{v}_1$, where $\bar{v}_1 = 1 - R_l$. In this case the expected profits function becomes $E(\pi_{PL}) = p(R_h - (1 - v_1)) - v_1$. Profits, $E(\pi_{PL}) \geq 0$, require $v_1 \leq \bar{v}_1 = \frac{p}{1 - p}(R_H - 1)$. In order for case 2 to hold we require, $\bar{v}_1 < \bar{v}_1$.

$$\frac{p}{1 - p}(R_H - 1) < 1 - R_l$$

$$pR_h + (1 - p)R_l < 1$$

$$\bar{R} < 1$$

Parameter value $\bar{R} < 1$ is not reasonable. Hence, $\bar{v}_1 > \bar{v}_1$ and profits are negative. QED.

References


**WORKING PAPERS**

00/01 Fernando C. Ballabriga, Sonsoles Castillo: BBVA-ARIES: un modelo de predicción y simulación para la economía de la UEM.

00/02 Rafael Doménech, María Teresa Ledo, David Taguas: Some new results on interest rate rules in EMU and in the US

00/03 Carmen Hernansanz, Miguel Sebastián: The Spanish Banks’ strategy in Latin America.

01/01 Jose Félix Izquierdo, Angel Melguizo, David Taguas: Imposición y Precios de Consumo.

01/02 Rafael Doménech, María Teresa Ledo, David Taguas: A Small Forward-Looking Macroeconomic Model for EMU

02/01 Jorge Blázquez, Miguel Sebastián: ¿Quién asume el coste en la crisis de deuda externa? El papel de la Inversión Extranjera Directa (IED)

03/01 Jorge Blázquez, Javier Santiso: México, ¿un ex - emergente?

04/01 Angel Melguizo, David Taguas: La ampliación europea al Este, mucho más que economía.

04/02 Manuel Balmaseda: L’Espagne, ni miracle ni mirage.

05/01 Alicia García-Herrero: Emerging Countries’ Sovereign Risk: Balance Sheets, Contagion and Risk Aversion

05/02 Alicia García-Herrero and María Soledad Martínez Pería: The mix of International bank’s foreign claims: Determinants and implications

05/03 Alicia García Herrero, Lucía Cuadro-Sáez: Finance for Growth: Does a Balanced Financial Structure Matter?

05/04 Rodrigo Falbo, Ernesto Gaba: Un estudio econométrico sobre el tipo de cambio en Argentina

05/05 Manuel Balmaseda, Ángel Melguizo, David Taguas: Las reformas necesarias en el sistema de pensiones contributivas en España.

06/01 Ociel Hernández Zamudio: Transmisión de choques macroeconómicos: modelo de pequeña escala con expectativas racionales para la economía mexicana

06/02 Alicia García-Herrero and Daniel Navia Simón: Why Banks go to Emerging Countries and What is the Impact for the Home Economy?


07/03  Ociel Hernández y Cecilia Posadas: Determinantes y características de los ciclos económicos en México y estimación del PIB potencial

07/04  Cristina Fernández, Juan Ramón García: Perspectivas del empleo ante el cambio de ciclo: un análisis de flujos.

08/01  Alicia García-Herrero, Juan M. Ruiz: Do trade and financial linkages foster business cycle synchronization in a small economy?

08/02  Alicia García-Herrero, Eli M. Remolona: Managing expectations by words and deeds: Monetary policy in Asia and the Pacific.

08/03  José Luis Escrivá, Alicia García-Herrero, Galo Nuño and Joaquín Vial: After Bretton Woods II.

08/04  Alicia García-Herrero, Daniel Santabárbara: Is the Chinese banking system benefiting from foreign investors?

08/05  Joaquín Vial, Angel Melguizo: Moving from Pay as You Go to Privately Manager Individual Pension Accounts: What have we learned after 25 years of the Chilean Pension Reform?

08/06  Alicia García-Herrero y Santiago Fernández de Lis: The Housing Boom and Bust in Spain: Impact of the Securitization Model and Dynamic Provisioning.

08/07  Ociel Hernández, Javier Amador: La tasa natural en México: un parámetro importante para la estrategia de política monetaria.

08/08  Patricia Álvarez-Plata, Alicia García-Herrero: To Dollarize or De-dollarize: Consequences for Monetary Policy

09/01  K.C. Fung, Alicia García-Herrero and Alan Siu: Production Sharing in Latin America and East Asia.

09/02  Alicia García-Herrero, Jacob Gyntelberg and Andrea Tesei: The Asian crisis: what did local stock markets expect?

09/03  Alicia Garcia-Herrero and Santiago Fernández de Lis: The Spanish Approach: Dynamic Provisioning and other Tools

09/04  Tatiana Alonso: Potencial futuro de la oferta mundial de petróleo: un análisis de las principales fuentes de incertidumbre.

09/05  Tatiana Alonso: Main sources of uncertainty in formulating potential growth scenarios for oil supply.

09/06  Ángel de la Fuente y Rafael Doménech: Convergencia real y envejecimiento: retos y propuestas.

09/08 Alicia García-Herrero, Philip Woolbridge and Doo Yong Yang: Why don’t Asians invest in Asia? The determinants of cross-border portfolio holdings.

09/09 Alicia García-Herrero, Sergio Gavilá and Daniel Santabárbara: What explains the low profitability of Chinese Banks?.

09/10 J.E. Boscá, R. Doménech and J. Ferri: Tax Reforms and Labour-market Performance: An Evaluation for Spain using REMS.


09/12 J.E. Boscá, R. Doménech and J. Ferri: Search, Nash Bargaining and Rule of Thumb Consumers

09/13 Angel Melguizo, Angel Muñoz, David Tuesta and Joaquín Vial: Reforma de las pensiones y política fiscal: algunas lecciones de Chile


09/15 Angel Melguizo, Angel Muñoz, David Tuesta and Joaquín Vial: Pension reform and fiscal policy: some lessons from Chile.

09/16 Alicia García-Herrero and Tuuli Koivu: China’s Exchange Rate Policy and Asian Trade


09/18 Alicia García Herrero y Daniel Santabárbara García; Una valoración de la reforma del sistema bancario de China

09/19 C. Fung, Alicia García-Herrero and Alan Siu: A Comparative Empirical Examination of Outward Direct Investment from Four Asian Economies: China, Japan, Republic of Korea and Taiwan

09/20 Javier Alonso, Jasmina Bjeletic, Carlos Herrera, Soledad Hormazábal, Ivonne Ordóñez, Carolina Romero and David Tuesta: Un balance de la inversión de los fondos de pensiones en infraestructura: la experiencia en Latinoamérica

09/21 Javier Alonso, Jasmina Bjeletic, Carlos Herrera, Soledad Hormazábal, Ivonne Ordóñez, Carolina Romero and David Tuesta: Proyecciones del impacto de los fondos de pensiones en la inversión en infraestructura y el crecimiento en Latinoamérica

10/01 Carlos Herrera: Rentabilidad de largo plazo y tasas de reemplazo en el Sistema de Pensiones de México

10/02 Javier Alonso, Jasmina Bjeletic, Carlos Herrera, Soledad Hormazabal, Ivonne Ordóñez, Carolina Romero, David Tuesta and Alfonso Ugarte: Projections of the Impact of Pension Funds on Investment in Infrastructure and Growth in Latin America

10/03 Javier Alonso, Jasmina Bjeletic, Carlos Herrera, Soledad Hormazabal, Ivonne Ordóñez, Carolina Romero, David Tuesta and Alfonso Ugarte: A balance of Pension Fund Infrastructure Investments: The Experience in Latin America

10/05 **Soledad Hormazabal D.:** Gobierno Corporativo y Administradoras de Fondos de Pensiones (AFP). El caso chileno.

10/06 **Soledad Hormazabal D.:** Corporate Governance and Pension Fund Administrators: The Chilean Case.

10/07 **Rafael Doménech, Juan Ramón García:** ¿Cómo Conseguir que Crezcan la Productividad y el Empleo, y Disminuya el Desequilibrio Exterior?

10/08 **Markus Brückner, Antonio Ciccone:** International Commodity Prices, Growth, and the Outbreak of Civil War in Sub-Saharan Africa.

10/09 **Antonio Ciccone, Marek Jarocinski:** Determinants of Economic Growth: Will Data Tell?

10/10 **Antonio Ciccone, Markus Brückner:** Rain and the Democratic Window of Opportunity.

10/11 **Eduardo Fuentes:** Incentivando la cotización voluntaria de los trabajadores independientes a los fondos de pensiones: una aproximación a partir del caso de Chile.

10/12 **Eduardo Fuentes:** Creating incentives for voluntary contributions to pension funds by independent workers: an informal evaluation based on the case of Chile.

10/13 **J. Andrés, J.E. Boscá, R. Doménech and J. Ferri:** Job Creation in Spain: Productivity Growth, Labour Market Reforms or both.

10/14 **Alicia García-Herrero:** Dynamic Provisioning: Some lessons from existing experiences.

10/15 **Arnoldo López-Marmolejo, Fabrizio López-Gallo Dey:** Public and Private Liquidity Providers.

The analyses, opinions and findings of these papers represent the views of their authors; they are not necessarily those of the BBVA Group.

BBVA Research disseminates its publications at the following website:
http://www.bbvaresearch.com