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Economic Watch

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Economic Analysis

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Growth, inflation, monetary rate, and interest rate curve: impact on both sides

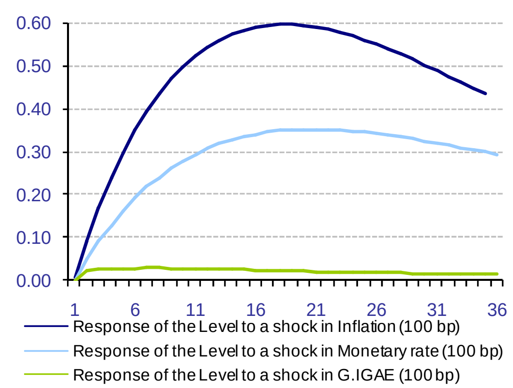
Estimation based on an *affine* model with macroeconomic variables of the temporary structure of interest rates in Mexico

- **Inflation, growth, and monetary policy affect the level and the slope of the rates curve, with the greater impact from inflation.**
- **Growth reacts negatively to the rise in interest rates.**
- **Finally, rates variations seem to anticipate monetary interest rate movements.**

In general, the yield curve structure—level, slope, and curve[1]—of Mexican public debt is affected by growth, inflation, and monetary policy. Inflation quickly affects the rates levels, and more significantly than the way growth does (chart a). In turn, the rates level and slope increase end up provoking a slowdown of growth (chart b).[2] On the short term, the rates level has a positive reaction to growth, as estimated by the IGAE (Global Indicator of Economic Activity). This is consistent with the close relationship of the rates level and inflation (charts a and 1), given that an increase in the level, which is in good measure an increase in inflation, would reduce the real rate, which would be followed by a short term growth boost. This pattern is also found in the US. The curvature does not have relevant impact on economic variables, although the latter do affect the curvature.

Chart a

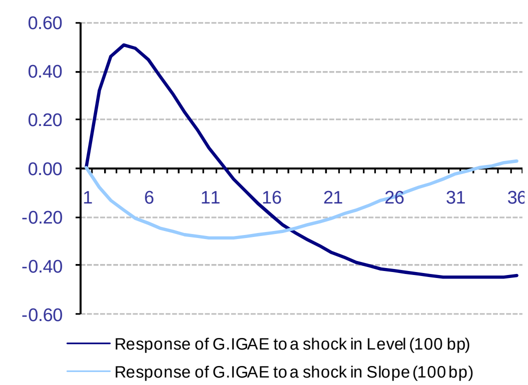
Reaction of the curve level to a change in inflation, monetary rate, and IGAE growth



Source: BBVA Research. G indicates growth. The numbers on the X axis are months. IGAE means Global Indicator of Economic Activity.

Chart b

Reaction of the growth of the IGAE to changes in the curve level and slope of 100 pbs



Source: BBVA Research. G indicates growth.

[1] The yields curve is simplified by modeling it using three factors that are interpreted as its level, slope, and curvature. Traditionally, the level of the curve is measured as the average of the yields of bonds that mature in the short, medium, and long term (e.g. the average of 3-month, 5-year, and 20-year bonds). The slope is the difference between the yield of a long-term bond and that of a short-term one (e.g. the difference between a 20-year bond and a 3-month one). The curvature refers to the difference between the yield of the bond in the medium part of the curve in relation to the short and long part (e.g. two multiplied by the yield of the two-year bond minus that of the 3-month and 10-year ones).

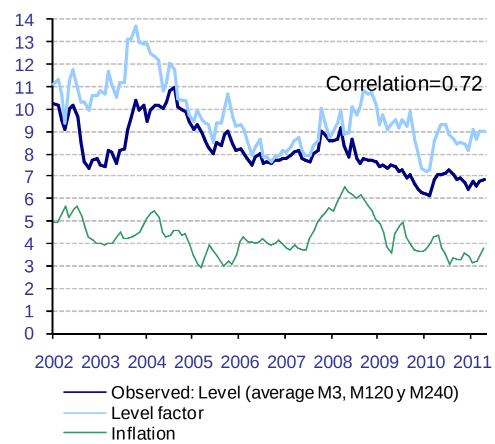
[2] The slope is defined as the negative of the "traditionally" defined slope ("long-term minus short-term").

Affine *model*: relationship between non-observable factors that define the rates curve and macroeconomic variables

This report analyzes the interaction between growth, inflation, and the monetary policy rate with the government bond yield curve, or more specifically with its level, slope, and curvature. This analysis is based on the Diebold, Rudebusch, and Boragan model (2006), a work that complements an *affine* model with three factors (level, slope, and curvature of the yield curve) with macroeconomic variables (in particular of the capacity used in the manufacturing sector, inflation, and monetary policy rate).[3] The model used is described in the appendix, and results of the estimates are shown. The intended purpose of the model is to attempt to ascertain the behavior of the entire yield curve, which is currently formed by bonds with maturity from 1 month to 30 years, through a mere three factors that synthesize it. These non-observable factors are interpreted, given their likeliness to them, as the level, the slope, and the curvature of the temporary structure of interest rates. The *affine* model seeks to explain the behavior of the three factors based on macroeconomic variables, while these are also affected by the level, slope, and curvature.[4]

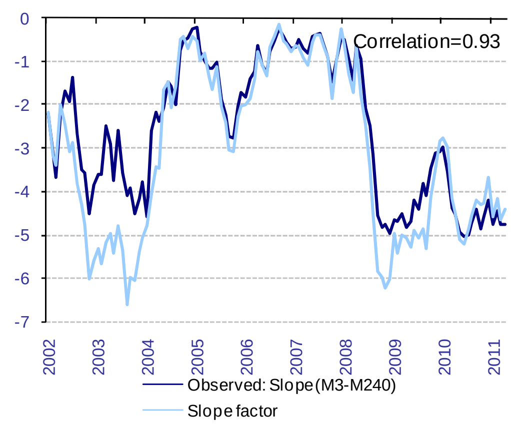
The model adequately captures the behavior of the level and the slope of the curve (charts 1 and 2). The correlation of the estimated factors of level and slope with the *proxy* levels observed is of 0.72 and 0.93, respectively. In the case of the level factor, if we consider as a *proxy* variable the yield of the ten-year bond, the correlation is 0.87.[5] In addition, a positive relationship is seen between the observed level and inflation (correlation 0.4).

Chart 1
Level: estimated factor and observed (percentage points)



Source: BBVA Research. To denote bonds, the letter M is used, referring to the month and the number on the right indicates maturity in months.

Chart 2
Slope: estimated factor and observed (percentage points)



Source: BBVA Research. The slope is defined as the negative of the "traditionally" defined slope ("long-term minus short-term").

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[3] Recently efforts have been made to analyze the yield curve in Mexico. For instance, Cortés and Ramos-Francia (2008a) developed an *affine* model for the level and slope. For the purpose of analyzing the relationship of rates with macro variables, Cortés, Ramos-Francia and Torres (2008) undertook an analysis of principal component to explain the level and slope of the curve. A study that makes it possible to incorporate the interaction of macroeconomic variables with the yields curve is that by Cortés and Ramos-Francia (2008b), in which the authors combine a macroeconomic model for a small and open economy with a temporary rates structure.

[4] In this work the structure of interest rates is analyzed, i.e. zero coupon rates. However, yields curve is also used to refer to this structure.

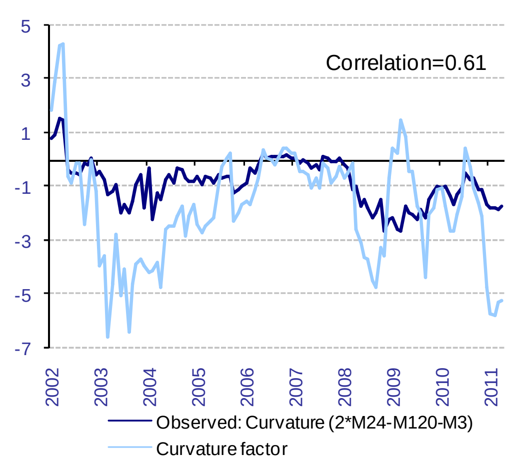
[5] The fact that in some periods there is a relevant difference between the estimated level factor and the *proxy* of the observed level suggests that other economic or financial variables may be relevant for certain periods of time. In particular, one that has resulted in the observed rate remaining lower than the estimated level.

As for the curvature, although the forecasted factor is more volatile than the observed level (chart 3), the tendency and its changing points seem to be adequately captured. Moreover, their correlation is higher than 0.6

The combination of the three components of the curve—level, slope, and curvature—together with the macroeconomic variables make a considerable adjustment in bonds of different maturities taken into account to build the curve. For instance, chart 4 shows the in-sample forecast and the level observed of the three-year bond. It can be seen that there are almost no errors.

Chart 3

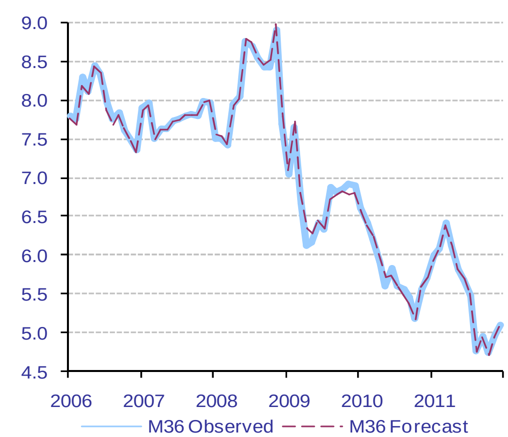
Curvature: estimated factor and observed (percentage points)



Source: BBVA Research.

Chart 4

Three-year bond yield: observed and forecast (percentage points)



Source: BBVA Research.

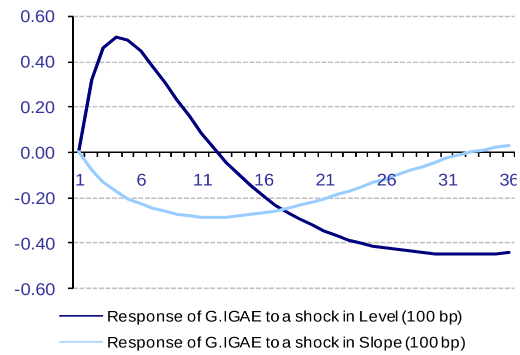
Effect of the temporary structure of interest rates on macroeconomic variables

It is estimated that there is an initial positive reaction of the growth of the IGAE (Economic Activity Index) to the curve level factor. However, this effect vanishes, becoming negative after approximately 12 months (chart 5), which appears to be consistent with the negative effect on economic performance of greater cost in the funding of agents. This same short-term positive reaction pattern of activity to the increase in the rates level is observed in the US by Diebold et al (2006), which they see as consistent given that the increase of the level is in great part because of increase in inflation, which reduces the real rate, boosting activity in the short term. As can be seen in chart 7, for the Mexican economy, it can also be seen that the level of nominal rates is highly and quickly affected by inflation changes.

With regard to an increase in the slope of the curve (relative increase in cost of short-term financing given that the slope is defined as short-term minus long-term), this seems to have a negative impact on growth (chart 5). This is consistent with greater financing costs in the short (monetary rate) and medium-term associated to increases in the slope (chart 6). A positive reaction of the monetary policy rate to changes in the level factor is also found. Given that there is more than one month between each monetary policy meeting, it is probable that yields react in anticipation to central bank actions, which the market can anticipate based for instance on the behavior of inflation, as can be seen in chart 7. In addition, as the Bank of Mexico has been communicating more effectively and taking foreseeable monetary policy decisions based on available economic information, bond market fluctuations should reflect a degree of anticipation of monetary policy. Finally, macroeconomic variables have quantitatively insignificant reactions to changes in the curvature factor (see table 2 in appendix).

Chart 5

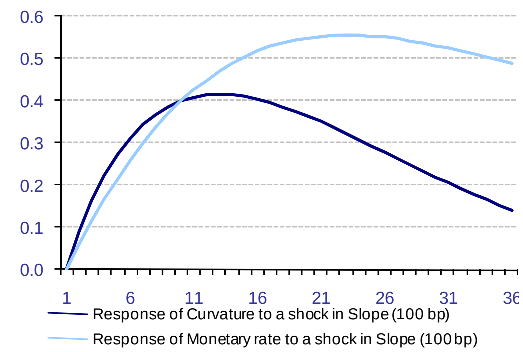
Reaction of the growth of the IGAE to changes in the level and slope factors of 100 bp



Source: BBVA Research.

Chart 6

Reaction of the curvature factor and monetary rate to a shock in the slope of 100 bp



Source: BBVA Research.

Effect of the macroeconomic variables on the temporary structure of interest rates estimated by level, slope, and curvature factors

The results obtained show that inflation has a very intense effect on the level and slope factors, positive and negative respectively (chart 7).[6] In keeping with these results, Cortes and Ramos-Francia (2008a) show that the level of the rates is closely linked to inflation expectations.

The response of the level and slope factors to inflation and monetary policy go in the same direction (charts 7 and 8). It may be that this occurs because the market is foreseeing that inflation increases are generally followed by increases in monetary policy rate.

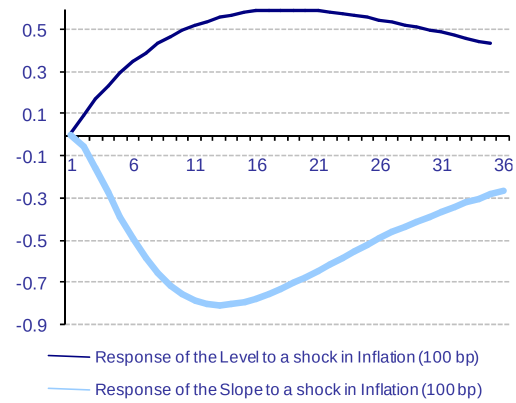
In addition, the negative effect of the monetary policy rate on the slope factor is in keeping not only with Cortes and Ramos-Francia (2008a) for Mexico but also with Rudebusch and Wu (2004) for the US.

Upward variations in the IGAE (Indicator of Economic Activity.) have a positive, although slight, effect on the level of the curve. The same takes place with the slope, which is why activity seems to increase more the short than the large part of the curve.

As for the curvature, growth and inflation tend to reduce it (chart 9), while monetary rate increases it (table 1 in appendix). It is possible that this takes place because the market expects that once the monetary policy rate has been changed, there is a high probability that the next move will be in the same direction.

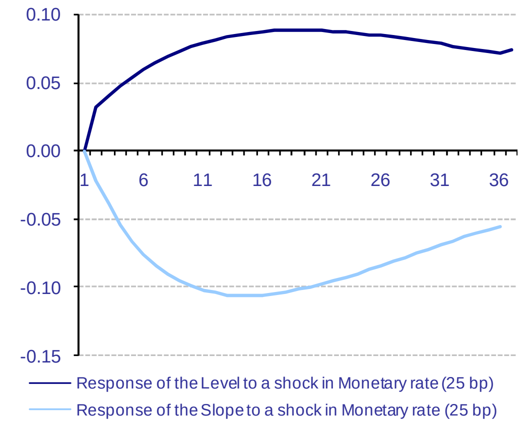
[6] It should be noted that the definition of slope is contrary to the habitual one, i.e., in this model it is the shorter term rate minus the longer one. Therefore, an increase in inflation raises the level of the curve and does so more significantly for longer-term rates than for shorter-term ones.

Chart 7
Reaction of the level and slope factors to a shock in yearly inflation of 1 pp



Source: BBVA Research. The slope is defined as the negative of the "traditionally" defined slope ("long-term minus short-term").

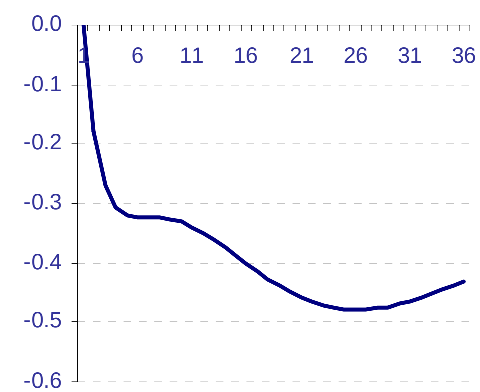
Chart 8
Reaction of the curve factor and lending to a shock in the curve of 25 bp



Source: BBVA Research.

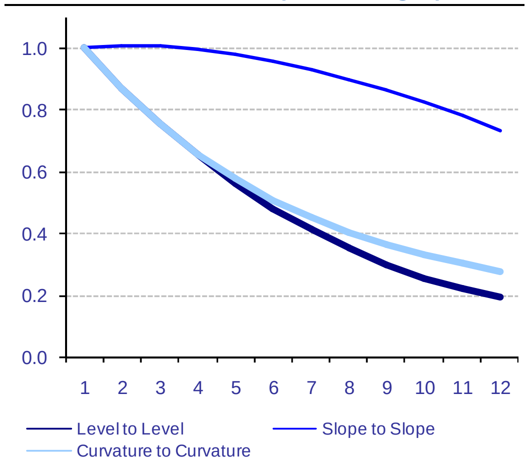
Finally, the strong persistence of level, slope, and curvature factors moderate the effects of the macroeconomic variables on the temporary structure of interest rates. The strong persistence is observed in chart 10, which shows that a change in the first period in the level, slope, and curvature factors does not disappear even after one year. To the extent that the persistence of these variables is reduced, the macro variables could bear greater influence on rates.

Chart 9
Reaction of the curve to an inflation shock of 1 percentage point



Source: BBVA Research.

Chart 10
Persistence: reaction of the level, slope, and measured curve factors to a shock in the curve itself of 1 percentage point



Source: BBVA Research.

Appendix

The period used for the estimation includes monthly data from September 2002 until December 2011. For simplicity's sake, the rates curve model without including macroeconomic variables is described first. So that it can be subsequently shown in a simple manner how these variables are incorporated into the model. It should be mentioned that the analysis refers to zero-coupon rates.

Representation of the unobserved factors model without macroeconomic variables

The factors model expresses the combination of the temporary interest rates structure for assets with varying maturities based on a small set of unobserved factors. Following Diebold et al. (2006), the set of interest rates is defined as $y(\tau)$, where τ indicates the maturity of the bond. A representation of the yields that has become very popular is the Nelson and Siegel curve (1987):

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right). \quad (1)$$

Where β_1. β_2, β_3 and λ are parameters. This representation may be interpreted in a dynamic form where β_1, β_2, and β_3 are factors that vary over time and represent the level, slope, and curvature. Therefore, the equation (1) can be re-written by substituting the betas by variables that change over time:

$$y(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right). \quad (2)$$

If L_t, S_t and C_t follow an auto-regressive process of order 1, the model is represented by a state-space system. The transition equation, which represents the dynamics of the state vector is given by:

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix}. \quad (3)$$

Where t=1,...,T. The measurement equation that relates the set of N interest rates with the three non-observable factors is:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_1) \\ . \\ . \\ . \\ y_t(\tau_1) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \varepsilon_t(\tau_N) \end{pmatrix}. \quad (4)$$

Notation with matrices makes it possible to write the state-space system as:

$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t$, (5)

$y_t = \Lambda f_t + \varepsilon_t$. (6)

Where $f_t´=(L_t, S_t, C_t)$. In order to obtain optimal results from minimal linear squares of the Kalman filter, errors must be white noise and orthogonal to initial conditions.

Expansion of the model incorporating macroeconomic variables

In this exercise it is sought to characterize the relationship between the level (L_t), the slope (S_t) and the curvature (C_t), with the macroeconomy. The variables of the economy chosen are three of its key indicators: economic activity growth ($G.IGAE_t$), the monetary policy rate (Monetary $rate_t$), and annual inflation ($Inflation_t$). These variables are widely accepted as the minimum set of fundamentals to represent the basic dynamics of the economy.

An extension of the model that considers solely yields adds three macroeconomic variables to the set of state variables. So that $f_t´=(L_t, S_t, C_t, G.IGAE_t$, Monetary $rate_t$, $Inflation_t)$, and the dimensions of A, μ, η_t, *and$_t$*, Λ, ε_t. increase accordingly.

Tables 1 and 2 show the estimated coefficients of the yields-macro model.

The growth of the IGAE (Global Indicator of Economic Activity.) has an effect that is more pronounced over the short part of the curve. The results show that the inflation has a positive effect, and is one of the most relevant factors for the level of the curve. The monetary policy rate is found to have a negative effect on the slope factor. This result is in keeping with that of Cortes and Ramos-Francia (2008) for Mexico and Rudebusch and Wu (2004) for the US.

Table 1

Determining factors for the level, slope, and measured curve of the temporary interest rates structure of government bonds in Mexico

	L_t ($Level_t$)	S_t ($Slope_t$)	C_t ($Curvature_t$)
L_{t-1}	0.93	0.06	0.03
	(0.02)**	(0.03)**	(0.08)
S_{t-1}	-0.03	0.99	0.08
	(0.02)*	(0.02)**	(0.05)
C_{t-1}	-0.03	0.07	0.85
	(0.02)*	(0.02)**	(0.05)**
$G.IGAE_{t-1}$	0.00	0.03	-0.07
	(0.01)	(0.01)**	(0.03)**
$Inflation_{t-1}$	0.11	-0.10	-0.23
	(0.03)**	(0.05)**	(0.13)*
Monetary $rate_{t-1}$	0.01	-0.04	0.12
	(0.01)*	(0.02)**	(0.05)**

Standard errors appear in parentheses.

** Significative at 5%; * Significative at 10%.

Source: BBVA Research.

In the case of the curvature, growth and inflation tend to reduce it, while monetary policy rate increases it. It is possible that this takes place because the market expects that once the monetary policy rate has been changed, there is a high probability that the next move will be in the same direction in the medium term.

The factors of the temporary structure of interest rates affect macroeconomic variables (table 2). In particular, the level has a positive initial impact on the growth of the IGAE, then reversing in the medium term. Furthermore, one can see a positive influence of the level on the monetary rate. Given that over a month passes between each monetary policy meeting, it is probably that the yields react in anticipation to central bank actions. Neither the level nor the slope seem to have any direct effect on inflation. The curvature has no effect on the analyzed macroeconomic variables.

Table 2
Factors determining macroeconomic variables in Mexico

	$G.IGAE_t$	$INFLATION_t$	$Monetary\ rate_t$
$G.IGAE_{t-1}$	0.68	0.01	-0.02
	(0.08)**	(0.01)	(0.01)*
$Inflation_{t-1}$	-1.33	0.95	-0.08
	(0.36)**	(0.04)**	(0.07)
$Monetary\ rate_{t-1}$	0.13	0.01	0.95
	(0.14)	(0.02)	(0.03)**
L_{t-1}	0.57	0.02	0.09
	(0.23)**	(0.02)	(0.04)**
S_{t-1}	0.02	0.02	0.08
	(0.16)	(0.02)	(0.034)**
C_{t-1}	0.02	0.01	-0.04
	(0.14)	(0.01)	(0.02)

Standard errors appear in parentheses.
** Significative at 5%; * Significative at 10%.

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