Economic Watch

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Economic Analysis

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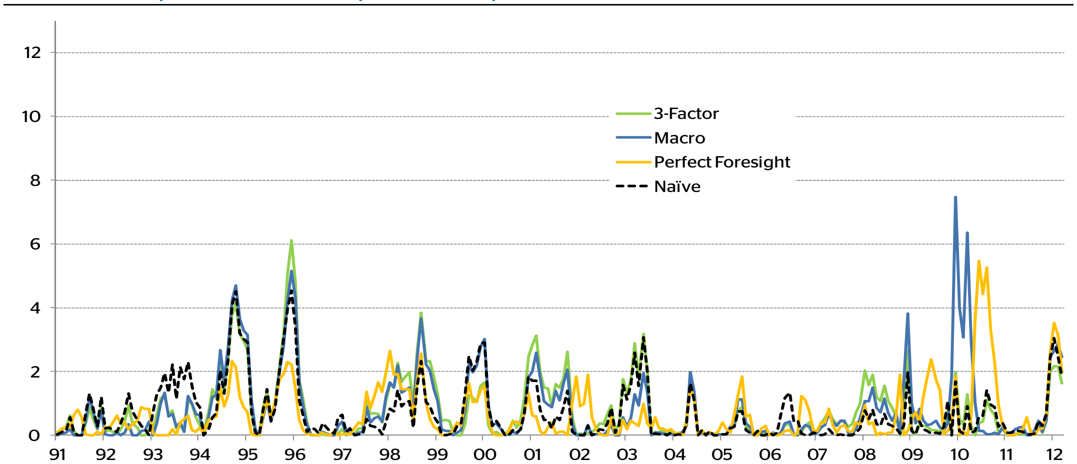
Forecasting Performance of Three-Factor Nelson-Siegel Models

Methods in Modeling and Forecasting the Term Structure

- **A simple three-factor model improves on a naïve 10-year yield forecast**
- **The addition of macro variables impedes forecasting performance in recent years, most likely related to unprecedented monetary policy interventions**
- **The macro factor model offers a robust means of forecasting the term structure conditional on economic scenarios**

The term structure of interest rates often represents a barometer of economic conditions. The yield curve comprises all information available as to the state of the economy. The sensitivity of bonds to interest rates, inflation and growth represents a means of capturing the behavior of the yield curve over time. This is due to the fact that bonds may be considered a type of interest rate derivative. While interest rates are volatile, we know that the returns of bonds of all maturities are perfectly correlated. We also know that the term structure, much like other derivatives, is uniquely determined by the level of the spot rate. Additionally, previous research has shown that three principal components explain most of the variation in yields composing the term structure. Creating a forecast for interest rates therefore involves capturing some of the effects of these factors. Term-structure modeling begins with a stochastic process of a single factor (for example, the short rate) or stochastic processes of multiple factors, such as the short-term interest rates and the yields of bonds of various maturities at any point in time. Another major component of the modeling scheme is to include assumptions about the existence of arbitrage in the term structure.

Chart 1
Model Mean Squared Error for 10-year Treasury Yield, 1990 to Present



Source: BBVA Research

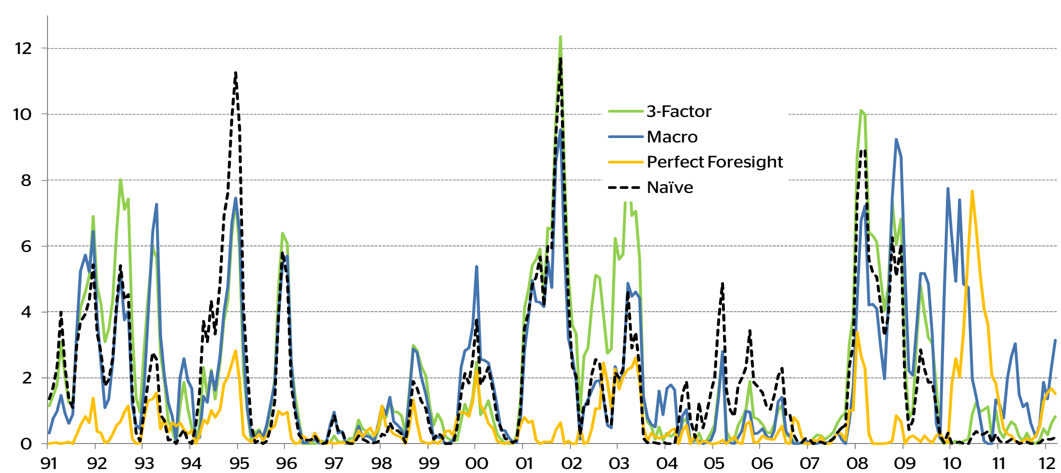
We can crudely divide models into two camps: Nelson-Siegel (NS) models and arbitrage-free (AF) term structure models. Drawing from the idea that the level, slope and curvature describe the term structure, NS models rely on at a minimum of three latent factors as parameters of mathematical approximating functions. NS models do not depend on the existence of arbitrage possibilities. The AF models draw heavily on the macro-financial literature and are based on particular equations of the diffusion process, such as Cox-Ingersoll-Ross. These models may depend on the absence of arbitrage opportunities and assume that the unobservable term structure factors follow stochastic processes. For the purposes of this brief, we will focus on NS-type models. The key to the NS approach is the following equation for the predicted yield at each time period for all maturities τ:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

The equation is specified such that yields at all maturities load identically on the first parameter and thus an increase in this parameter increases all yields equally. Short rates load more heavily on the second parameter than long rates and thus a change in the second parameter affects the slope. Lastly, the third parameter loads heavily on the middle of the yield curve. The parameter lambda controls both the exponential decay rate and the maturity at which the loading on the curvature reaches its maximum. When introduced into a state space format, the latent factors may be related to macroeconomic variables. Additionally, a one-step estimation procedure may be used with Kalman filter techniques, which provides maximum-likelihood estimates and optimal filtered and smoothed underlying factors.

We take each part of the term structure and evaluate the forecasting performance of three different NS-type models: a Diebold Li (2006) model, a Diebold Rudebusch Aruoba (DRA 2006) and what we are calling a "perfect foresight" Diebold Rudebusch Aruoba model. The Diebold Li (DL) specification uses a three-factor NS specification without macroeconomic variables to guide the factors. Our DRA model takes the NS-type and adds on macroeconomic variables (the YoY CPI, Fed Funds rate, and YoY industrial production) to guide the factors. We use the actual macroeconomic data to estimate the factors through a particular month; however, we allow the Kalman filter to produce internal forecasts of the macro variables over the different horizons. Our perfect foresight model (DRAPF) is the DRA specification but instead of model-generated forecasts of macro variables, we use the realized macroeconomic data over the chosen horizon to forecast the factors. This is to determine how the DRA model would perform if it had 100 percent accurate knowledge of the future movement of inflation, the Federal Reserve and industrial production. With regard to lambda, we also let the Kalman filter determine the optimal setting for this parameter, while the original authors fix this parameter beforehand. Sensitivity to lambda is a topic we will explore in a future brief.

Chart 2
Model Mean Squared Error for 2-year Treasury Yield, 1990 to Present



Source: BBVA Research

We next test the out-of-sample forecasting performance of these three methods at different forecasting horizons of 3, 6 and 12 months and examine the RMSE and Theil's U statistics. The Theil U statistic compares the ratio of the model's RMSE to the RMSE of a naive forecast of no change in the yield. We examined these statistics over three time periods: Jan 1990-Jan 2000, Jan 2004-June 2007, Jan-2008-March 2012. For example, we estimate the model up until 1990 and then as each data point is added we evaluate the unfolding forecast performance. We define the forecast error and the Theil U statistic for each maturity as:

$$\hat{\varepsilon}_{t+h,i} = \hat{y}_{t+h,i} - y_{t+h}$$
$$\hat{\varepsilon}_{t+h,naive} = y_t - y_{t+h}$$
$$U_{i,naive,h} = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\varepsilon}_{t+h,i})^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\varepsilon}_{t+h,naive})^2}}, \quad \forall\, i \in \{3\ Factor, Macro, Perfect\ Foresight\}, \quad \forall\, h \in \{3, 6, 12\}$$

Where t refers to a particular month, h is the chosen horizon at which we evaluate the forecast performance, y_t is an actual yield at time t, and $\hat{y}_{t+h,i}$ represents the forecast of a yield at time $t+h$ for model i. Beginning with the 1990s, we find that the macro factor model improved marginally on the forecast of the yield curve at the 6- and 12-month horizons. The perfect foresight model, however, dramatically reduced the forecast errors at the 6- and 12-month horizons. Turning to the 2004 evaluation period, both the three factor and macro factor models improve upon the forecast at all horizons, but the perfect foresight model struggles to improve the 6-and 12-month ahead forecast of the 7- and 10-year yields. Interestingly, the three factor model alone performs the best for those yields at the 6 and 12 month horizons. During this time period, the slope of the yield curve was flat for a long period of time and Federal Reserve researchers wondered if there was an "interest rates conundrum" related to the secular decline in the term premium. This quandary may or may not be related to the concurrent global transfer of investments to the US during its housing boom.

Table 1
Theil's U Statistics by Model, Horizon and Maturity for 1990:1 to 2000:1 Evaluation Period

	12-month ahead			6-month ahead			3-month ahead		
Maturity (months)	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight
3	1.19	1.08	0.33	1.32	1.12	0.57	1.64	1.29	0.93
6	1.20	1.11	0.40	1.31	1.16	0.65	1.50	1.25	0.95
12	1.07	1.03	0.45	1.07	1.07	0.71	1.05	1.06	0.96
24	1.04	1.02	0.54	1.05	1.06	0.76	1.04	1.07	0.98
36	1.03	1.00	0.58	1.03	1.04	0.77	1.03	1.06	0.96
48	1.02	1.00	0.63	1.02	1.02	0.78	1.03	1.04	0.93
60	1.01	0.99	0.67	1.02	1.01	0.78	1.02	1.03	0.91
84	1.01	0.99	0.74	1.02	1.01	0.81	1.03	1.02	0.91
120	1.01	1.00	0.82	1.03	1.02	0.86	1.04	1.04	0.94

Source: BBVA Research

Table 2
Theil's U Statistics by Model, Horizon and Maturity for 2004:1 to 2007:1 Evaluation Period

	12-month ahead			6-month ahead			3-month ahead		
Maturity	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight
3	0.47	0.39	0.15	0.43	0.43	0.30	0.93	0.81	0.67
6	0.54	0.46	0.17	0.48	0.52	0.30	0.71	0.70	0.55
12	0.64	0.56	0.28	0.66	0.72	0.53	0.76	0.89	0.85
24	0.66	0.57	0.37	0.74	0.79	0.67	0.87	0.95	0.93
36	0.65	0.57	0.50	0.77	0.81	0.76	0.90	0.96	0.94
48	0.66	0.61	0.66	0.80	0.84	0.86	0.91	0.96	0.94
60	0.71	0.68	0.84	0.86	0.89	0.95	0.94	0.96	0.95
84	0.87	0.89	1.14	0.96	0.98	1.05	0.98	0.98	0.95
120	1.00	1.04	1.29	0.98	0.99	1.03	1.00	1.00	0.94

Source: BBVA Research

Table 3

Theil's U Statistics by Model, Horizon and Maturity for 2008:1 to 2012:3 Evaluation Period

	12-month ahead			6-month ahead			3-month ahead		
Maturity	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight	3-Factor	Macro	Perfect Foresight
3	1.17	2.63	0.87	1.27	2.24	0.83	1.16	1.66	0.84
6	1.11	2.40	0.97	1.16	2.10	0.89	0.99	1.58	0.82
12	1.23	2.43	1.29	1.12	1.90	1.05	1.05	1.55	1.00
24	1.41	2.36	1.78	1.13	1.67	1.26	1.06	1.38	1.09
36	1.43	2.19	1.93	1.12	1.52	1.29	1.05	1.29	1.11
48	1.36	1.99	1.89	1.09	1.42	1.28	1.04	1.23	1.11
60	1.26	1.82	1.80	1.06	1.34	1.24	1.03	1.19	1.10
84	1.08	1.57	1.63	1.01	1.24	1.19	1.01	1.13	1.08
120	0.94	1.40	1.52	0.98	1.17	1.17	1.02	1.11	1.11

Source: BBVA Research

With regard to the most recent evaluation period starting in Jan 2008, while all of the models encounter trouble improving on a naïve forecast of no change, the RMSE of the 10-year yield forecast from the three factor model is lower than that of a naïve forecast at the 6- and 12-month ahead horizons. Part of the issue during this time period is that our macro factor model is encountering difficulty forecasting under zero bound conditions. For example, in 2009, the model predicts negative yields at the short-end of the yield curve. The perfect foresight model improves upon the short-end of the yield curve since in this model the path of the Fed Funds rate is known into the future (and hence nonnegative).

Bottom line: Lessons Learned

We examined the forecasting performance of three-factor NS models during different time periods. Our analysis confirms that these models are able to capture the changing shape of the yield curve over all our selected time periods. Medeiros and Rodriquez (2011) review these same types of models and also find that DL and DRA models capture well the evolution of the yield curve since the crisis. The models pick up on the downward shift in the term structure, the flattening of the slope and a later reversal of the slope and decline in curvature. Our results, however, suggest that while these factors still behave as expected, the predictive power of a model augmented with macro factors (the Fed Funds rate, inflation and industrial production) appears to be less relevant in the current environment: based on data outturns, term structure models with macro variables tend to over-predict yields, particularly at the long end of the yield curve. Notably, the past few years have experienced unprecedented monetary intervention such as quantitative easing and other attempts to flatten the yield curve. This result contrasts with the strong performance of these macro-factor models over 1990-2000 during a period of conventional monetary policy. In a following brief, we will explore the usefulness of different macro factors in these types of models along with additional methods of forecasting the yield curve.

References

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