

Working Paper
Number 13/15

Projections of dynamic generational tables and longevity risk in Chile

Economic Analysis
Madrid, April 8, 2013

Projections of dynamic generational tables and longevity risk in Chile

Authors: Javier Alonso y David Tuesta, Diego Torres, Begoña Villamide

March 07, 2013

Abstract

The increase in longevity risk is leading to serious challenges for economies. Industries such as insurance and pensions, which are most closely related to the management of the risks of an aging population, have for a number of years experienced direct effects of this kind. To counterbalance this, they have developed techniques for constructing mortality tables in order to project the future trends of life expectancy at birth and thus reduce the level of uncertainty that this market by its nature involves. Developed countries have led technical improvements for constructing these tables, while Latin American countries have lagged behind significantly in this respect. Given that these countries cannot yet develop tables weighted by social and medical aspects, it is highly probable that this situation will continue. That is why this study aims to construct a forecast for mortality rates, based on projection models of the ARMA (p, q) type and non-parametric contrast methodology. The study is based on the case of Chile, which provides most information for constructing a model for a Latin American country. The estimates show that the official mortality tables in Chile could include significant lags by 2050, which will have major negative effects on the pension and insurance industry, in the hypothetical case that they were not updated. In another exercise, using the mortality table estimated in this work, we found that if pensions in Chile are not to lose their purchasing power, the contribution rate would have to be increased by 8 percentage points in the case of men and 4 in the case of women. Given that Chile is the best developed country in the region with respect to mortality tables, the negative effects on the rest of Latin America could be even more worrisome.

Key words: Pensions, insurance, longevity risk, mortality tables, Latin America, Chile.

JEL: G23, J32, G22.

1. Introduction

In the second half of the last century, the world began to show clear signs of major demographic changes that would affect the way labor markets and, in the final analysis, the economy itself, behaved. One of these changes is the greater longevity of life expectancy. As progress has been made in health, nutrition, government health policies and lifestyles, life expectancy has increased substantially. At the start of the 20th century the average life expectancy was barely over 30, and was below 50 even among the highest-income groups (Prentice, 2006). As a consequence of that, this average increased to 48 years mid-way through the century, and currently is estimated to be over 66 years, with a figure of over 80 in some countries.

Although these average figures still show major contrasts between different geographical areas, particularly between the so-called developed and lower-income countries, it is also true that a very swift process of convergence is underway as the social and medical progress spreads. In fact, in the last 30 years the number of people who live to 60 years or over has doubled and it is expected to double again by 2050, when this age group will be larger than the population under the age of 14.

The transition to an economic reality with longer-living populations involves important challenges for the economy as a whole, although in a particularly direct way for those industries that have focused their objective on the coverage of risks associated with old age, such as pensions and insurance. The "risk" of living longer does not appear to have reached its limit yet, so it incorporates an element of uncertainty with respect to gauging the products offered correctly. A key element when it comes to designing insurance products (and thus adjusting their respective prices) to address the risk associated with inactive old age, is to have well designed mortality tables with projections that can provide the best possible balance between the needs of buyers and sellers.

Developed countries were the first to note the pressures from an aging society, which began to make inroads first on the public finances, and then on the private insurance industries themselves. However, developing countries are converging at an increasingly fast pace. In the 1950s, the gap between life expectancy at birth for these two groups of countries was around 20 years; it has now been narrowed to around 5 years. The richest countries were the first to make significant innovations in the design of mortality tables, and they were updated constantly. The most advanced countries such as Germany even include social and bio-medical information to fine-tune their estimates. Latin America has lagged behind this progress. It could be said that only in the last decade have serious steps been taken to improve the tables, with the inclusion of dynamic tables. Chile has taken the lead in this respect. Nevertheless, they still lag behind significantly because of the methodologies used, which cannot include the whole range of factors that interact in the projections of mortality tables.

With this in mind, this paper includes an econometric analysis designed to reveal the potential gap produced by the mortality tables currently used in Latin America. For this purpose we have focused on the case of Chile, because it is the country with most information available for making the necessary calculations. In general, the problems found in the study suggest that the challenges that could be facing other mortality tables in the region could be bigger, bearing in mind that Chile is the country that has worked most seriously and for the longest time on this subject.

A difficulty for Latin American countries in general, and Chile in particular, is that they do not have sufficient past information on mortality trends to construct their own long-term tables. One way of dealing with this is to compare the information available in these countries' tables with that of other countries for which we have more information. We can use sample equivalence tests to identify those that offer the greatest similarities.

In our case, the sample equivalence analysis uses non-parametric tests that compare the level of mortality (sign test), form (runs test) and the existence of a similar pattern between both tables (χ^2 test). The application of these tests will allow us to determine which country has mortality tables that are statistically comparable to the Chilean, N years ahead/behind. The study also includes the additional condition that the equivalence of the tables must be stable and prolonged over time, to ensure that this equivalence is not limited to a chance episode in time. The model projects the mortality rates in Chile using the ARMA (p, q) methodology for each of the 101 ages and for each gender, with a total of 202 regressions.

As will be seen in the study, there are significant differences between the official tables and those given by our estimates. Finally, the work calculates the economic impact of the use of inadequate mortality tables.

After this introduction, the second section reviews how far Latin America has progressed in the development of mortality tables. The third section discusses different types of methodologies for projecting mortality tables. Section 4 presents the estimates made to calculate the projected mortality tables for Chile and compare them with those currently used. Section 5 demonstrates the economic effects of using inadequate mortality tables; and finally, the last section presents the main conclusions of the study.

2. Mortality tables in Latin America and longevity risk

Demography is the science that studies human populations, their characteristics and their dynamics. The first works to deal with statistical demographic data were developed by Ibn Khaldun (1332-1406). However, the science only finally took off in the United Kingdom with the works of John Graunt in the 17th century and Robert Malthus in the 18th.

Population censuses are the main source of demographic information. Although censuses have been held in various cultures for thousands of years with the aim of making tax collection easier, modern censuses arose at the start of the 19th century in European countries such as the United Kingdom and Sweden. The biggest census project in Latin America was launched when the Inter-American Statistical Institute (IASI) launched its Census of the Americas (COTA) program, which was followed a large number of countries in the region in 1950. The delay in the availability of statistics, combined in many cases the need for better technical skills on the part of specialists in some countries, has led to a lag in the availability of information such as mortality tables that are needed to develop actuarial science.

In Chile, until 2004 the RV-85 mortality tables were used, as established by Circular No. 656 of the pension manager supervisor Superintendencia de Administradoras de Fondos de Pensiones (AFP)¹. These tables were originally designed to be applied to the US population, but were adjusted using CELADE data from the 1982 census.

However, starting in 2000 the RV-85 tables began to show signs of being out of date for the calculation of life expectancy. This deficiency led to the AFP Supervisor (Pension Supervisor) and Supervisor of Securities and Insurance to construct the RV-2004 mortality tables. The new tables updated the calculation of the programmed retirement benefits and life annuities using the data of retirement pensioners for the period 1995-2003, inclusive. Using adjustment and graduation techniques, they determine the probable life expectancy of future pensioners. The most notable result is that the RV-2004 tables showed the RV-85 had underestimated life expectancy. For example, the life expectancy of women aged 60 increased by up to 3.06 years with the new tables. In the case of men the error was not as large, at around 0.51 years below the real figure.

¹ For more details, see: <http://www.spensiones.cl/redirect/files/normativa/circulares/CAFP656.pdf>, <http://www.safp.cl/safpstats/stats/files/normativa/circulares/CAFP1314.pdf>, and the press release: http://www.svs.cl/comunicados/com_20041116-01.pdf.

Colombia had been using the same mortality table for pensioners since 1994 (RV89, based on the data for 1989). In 2000, the Colombian banking supervisor Superintendencia Bancaria de Colombia issued Circular 071/2000² requesting information on the mortality of active contributors, pensioners, the disabled and people with invalidity, with the aim of preparing up-to-date mortality tables. An analysis of these data revealed that these tables had to be updated, and in 2010 Resolution 1555/2010³ resolved to replace the RV89 with the RV08 tables.

As in the case of Chile, the updated tables in Colombia revealed that life expectancy had been underestimated. The RV08 calculated a life expectancy for women aged 60 of 27 years, while the calculations of the National Administrative Department of Statistics (DANE) put the figure at around 22, a difference of 5 years. In the case of men, the difference was not as great, at 3.3 years.

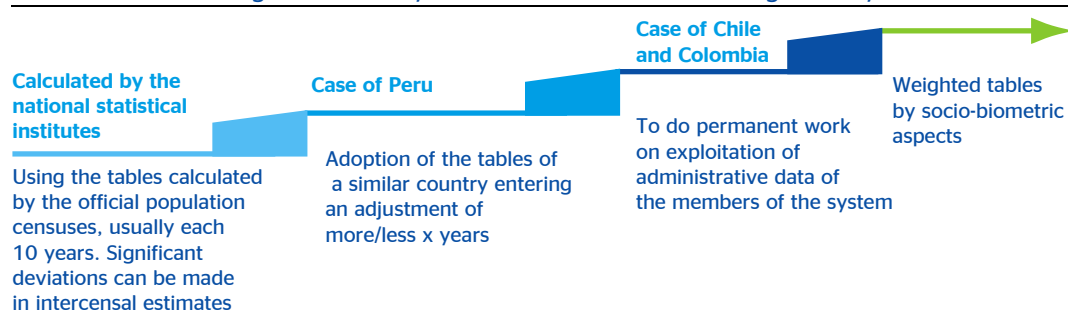
Peru had since 1993 used the Chilean RV85 tables, but after 13 years, in 2006, Resolution 354/2006 approved the use of modified Chilean RV-2004 tables. More recently, Resolution 17728/2010 included as obligatory the use of RV-2004 tables modified and adjusted to the Peruvian experience⁴.

The mortality tables used to calculate life annuities in the Mexican social security system are prepared by the National Insurance and Finance Commission, which is the regulator and supervisor of pension insurers. From 1997 to 2008 they used mortality tables called EMSSA 97 (Mexican Social Security Active Experience), based on a study projecting mortality prepared by the National Population Council (CONAPO) for the period 1990-2030. They were broken down by gender, but did not take into account improvements in mortality.

In 2007 a statistical analysis of the observed mortality of pensioners with life annuities was carried out for the period 1997-2006. This showed a greater survival rate than expected. As a result, in 2009 it was decided to adopt the dynamic mortality tables called EMSSA 2009, which take into account improvements in forecast life expectancy for the coming years, in accordance with the demographic trend for increased longevity, and using the study of projected mortality in Mexico for the period 2005-2050 carried out by CONAPO.

Chart 1 sums up the different methodologies used to calculate mortality tables in Chile, Peru, Colombia and Mexico. Chilean and Colombian tables are clearly better positioned, against an ideal benchmark of methodologies for tables based on social and bio-medical aspects, which would ensure a better estimate of life expectancy and its future development.

Chart 1
Calculation methodologies of mortality tables in Latin America at the age of 65 years old



Source: BBVA Research

2: For more details see Circular 071/2000:
<http://www.superfinanciera.gov.co/Normativa/PrincipalesPublicaciones/boletinminhda/2000/548-1000/ce071.rtf>
3: http://www.superfinanciera.gov.co/NormativaFinanciera/Archivos/r1555_10.doc
4: For more details see the following resolutions:
<http://intranet1.sbs.gob.pe/IDXALL/SEGUROS/DOC/RESOLUCION/PDF/0309-1993.R.PDF>
<http://intranet1.sbs.gob.pe/IDXALL/SEGUROS/DOC/RESOLUCION/PDF/0354-2006.R.PDF>
<http://intranet1.sbs.gob.pe/IDXALL/SEGUROS/DOC/RESOLUCION/PDF/17728-2010.R.PDF>

3. Projection methodologies for mortality tables

The first works to project mortality can be traced back to 1875, when the Swiss astronomer H. Gylden fitted a straight line over a sequence of general mortality rates of the Swiss population for the period 1750-1870 and then extrapolated it as a projection (Pitacco et al., 2009). Following the classification of models used by Camarda (2008) we can mainly distinguish parametric and non-parametric models.

3.1 Models for projecting mortality tables

We will now briefly discuss each of the models used to project mortality tables. However, we first have to define some general measurements of mortality that will be used later:

Let us define X as a random non-negative and continuous variable that describes the future life span of individuals. Three functions characterize and describe the distribution of X : the function of probabilistic density, the function of survival and the risk rates (or mortality risk rates).

The survival function is defined as the probability that an individual will survive more than x years, using the formula:

Equation 1

$$S(x) = \Pr(X > x) = \int_x^{\infty} f(t) dt$$

Equation 1 is in turn the complement of the cumulative distribution function, i.e.:

Equation 2

$$S(x) = 1 - F(x)$$

Using Equation 1 and Equation 2 we get

Equation 1

$$f(x) = -S'(x)$$

Another fundamental function is the "risk function", which measures the probability that a certain event may occur to an individual over time; in our case it is the probability that an individual dies.

The probability that an individual over the age of 35 will not live longer than 75 years is given by the following conditional probability:

$$\Pr(35 < X \leq 75 | X > 35) = \frac{F(75) - F(35)}{S(35)}$$

In general terms, the outcome probability (risk function) is the limit of the conditional probability when the interval becomes infinitely small, i.e.

Equation 4

$$h(x) = \lim_{\Delta x \rightarrow 0} \frac{\Pr(x < X \leq x + \Delta x | X > x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \left(\frac{F(x + \Delta x) - F(x)}{S(x)} \right) = \frac{f(x)}{S(x)} = -\frac{d \ln S(x)}{dx}$$

The last equivalence is given using Equation 3 and the derivation rule for logarithmic functions.

Using Equation 4 we can derive an identity that relates the survival function $S(x)$ to the risk function $h(x)$

Equation 5

$$\begin{aligned} h(x) = -\frac{d \ln S(x)}{dx} &\Leftrightarrow -\int h(u) du = \int \frac{d \ln S(u)}{du} du \\ &\Leftrightarrow -\int h(u) du = \ln S(x) \Leftrightarrow \exp\left[-\int h(u) du\right] = S(x) \end{aligned}$$

$$S(x) = \exp[-H(x)]$$

Equation $H(x) = \int h(u) du$ is called the cumulative risk function.

Next we will describe each of the models.

3.2 Parametric models:

3.2.1 Age-based models

Gompertz (1825): Gompertz was one of the first to model mortality as a level of risk. He observed that in the age range of 30 to 80, mortality risk increased exponentially as age increased. He therefore suggested that the level of risk should be modeled as follows:

$$h(x) = a \cdot e^{b \cdot x}$$

Where x is the age of the individual with relation to a baseline age, $a > 0$ is the mortality rate at age "0" (generally 30 years), and $b > 0$ is the rate of increase in mortality per extra year of life.

Using Equation 3 and Equation 5 we can derive the density function for the Gompertz distribution.

Equation 6

$$f(x) = a \cdot e^{b \cdot x} \exp\left[-\int a \cdot e^{b \cdot u} du\right] = a \cdot e^{b \cdot x} \exp\left[-\left(\frac{a(e^{bx} - 1)}{b}\right)\right]$$

Equation 7

$$f(x) = a \cdot e^{b \cdot x} \exp\left[\left(\frac{a(1 - e^{bx})}{b}\right)\right]$$

Makeham (1860): Makeham extended Gompertz's equation by including a constant that absorbed the mortality risk that is independent of age:

$$h(x) = c + a \cdot e^{b \cdot x}$$

In this case as well, following the same procedure as above, we can derive the probability density function:

Equation 8

$$f(x) = a \cdot e^{b \cdot x} \exp \left[\left(-cx + \frac{a(1 - e^{bx})}{b} \right) \right]$$

This model, as in the case of Gompertz, only models mortality for adult ages. The models presented below aim to model the special characteristics of all ages, i.e. the high rate of infant mortality, which falls drastically in the initial years of life; a bottoming-out during youth and adulthood and an upturn in mortality after 80 years.

The models of Perks (1932), Thatcher (1999): logistic models have been among the main models proposed for the behavior of mortality rates at ages of 80 and over. Perks was the first to propose a logistic modification to the Gompertz-Makeham models. The modified equation is:

Equation 9

$$h(x) = c + \frac{a \cdot e^{b \cdot x}}{1 + \alpha e^{b \cdot x}}$$

As we can see, this function covers that of Makeham (when $\alpha = 0$) and that of Gompertz (when $\alpha = 0$ and $c = 0$). The model proposed by Thatcher is similar to that of Perks.

3.2.2 Models that describe the behavior of all age groups

Heligman and Pollard (1980) have constructed a model that aims to describe the behavior of the mortality rate in all age groups:

Equation 10

$$h(x) = A^{(x+B)^C} + D e^{-E(\ln x - \ln F)^2} + \frac{GH^x}{1 + GH^x}$$

Where A, B, ..., H are the parameters of the model. This parametrization makes the calculation difficult, and it is also difficult to give a meaning to the estimated parameters.

Siler (1983) proposed a model that integrates 3 models of mortality to describe the behavior of all the age groups with 5 parameters. Anson (1988) proposed a fifth-degree polynomial to represent the mortality risk rates in humans.

3.2.3 Age and time-based models

The above models only captured the fact that the mortality rate will change as age increases. In this section, we will comment the models that have been developed to include the time dynamic of mortality rates

3.2.3.1 Relational models

These models relate a reference (standard) mortality rate, for example that obtained using the Gompertz model, to the mortality rate of the population at time "t" and a specific age.

Himes et al. (1994) proposed a relational model of the type:

$$Y_t(x) = \delta + \sum_x \beta_x I_x + \sum_t \gamma_t J_t$$

Where $Y_j(x)$ is the logistic transformation of the mortality rates for each age x , δ is the component of mortality independent of age and time, I_x is a variable dummy for age x , J_t is a variable dummy for the year analyzed, β_x is the age-specific mortality rate that is independent of the passage of time, γ_t is the mortality rate corresponding to time (such as due to progress in questions of health).

3.2.3.2 The Age-Period-Cohort Model (APC)

These models are developed with the aim of separating out changes in the mortality rate due to three demographic coordinates: age, period and cohort.

Mathematically this model suggests that the natural logarithm of the mortality rates may be represented as follows:

Equation 11

$$\underbrace{\ln(m_{xt})}_{\substack{\text{Logaritmo} \\ \text{natural} \\ \text{Tasa de} \\ \text{Mortalidad}}} = \underbrace{\alpha_x}_{\substack{\text{Efecto} \\ \text{edad}}} + \underbrace{\beta_t}_{\substack{\text{Efecto} \\ \text{tendencia} \\ \text{temporal}}} + \underbrace{\lambda_c}_{\substack{\text{Efecto} \\ \text{Cohorte}}} + \underbrace{\varepsilon_{xt}}_{\substack{\text{Efecto} \\ \text{edad y tiempo}}}$$

$$x = 1, \dots, m$$

$$t = 1, \dots, T$$

$$c = 1, \dots, m + T - 1$$

The problem with this model is that of the total $2m+2n-1$ parameters to be calculated (in other words, m parameters for age, plus T parameters for time and $m+T-1$ parameters for the cohorts), only $2m+2n-4$ are identifiable (if we estimate $m-1$ parameters of age, the m can be obtained as a linear combination of the others, as is the case with time and the cohort, so we have 3 parameters that are linear combinations of the previous ones).

3.2.3.3 The Lee-Carter model

Lee and Carter (1992) reduced the complexity of the ATC model presented above and transformed it into a bilinear one (i.e. adding a multiplicative interaction between the demographic dimensions that affect mortality).

Equation 12

$$\ln(m_{xt}) = \alpha_x + \beta_x \lambda_t + \varepsilon_{xt}$$

$$x = 1, \dots, m$$

$$t = 1, \dots, T$$

The variance of ε_{xt} is assumed constant for all ages and periods (this assumption is relaxed in some variants of this model).

This model requires additional restrictions for its calculation. Commonly $\alpha_x = \frac{1}{n} \sum_{t=1}^T \ln(m_{xt})$ is imposed, so we focus the model on the average mortality rates for the period under analysis. Using this restriction, we can interpret that β_x represent the fixed effect of age or deviations with respect to the mean observed over time. λ_t is a mortality rate that varies over time.

This is one of the most commonly used models for a variety of demographic purposes and it is considered as the standard model for modeling and predicting mortality rates.

What is attractive about the model is that the mortality rate obtained λ_t sums up the "global" trend. The procedure used for the projection is as follows:

- 1) Calculate $\alpha_x, \beta_x, y \lambda_t$
- 2) Model λ_t as an ARIMA process
- 3) With the estimated ARIMA model, project the values of λ_t
- 4) Use the estimated values of $\alpha_x, y \beta_x$ together with the projections of λ_t to obtain the projected mortality rates.

As we will see in the following section our model could be classified within this last group (parametric age-period models).

4. Description of the methodology applied in this study

For the projection exercises, the mortality rates were estimated using ARMA(p, q) models for each of the 101 ages and for each gender, with a total of 202 ARMA(p,q)regressions:

Equation 13

$$(1 - \Phi_1 B - \Phi_2 B^2 - \dots - \Phi_p B^p) y_t = (1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q) u_t + \alpha + \beta t + \lambda t^2 + \delta t^3$$

Where y_t represents the Naperian logarithm of the mortality rate for the population of a certain age and sex, B is the lag operator ($B y_t = Y_{t-1}$), t represents the time trend and u_t is the part not captured by the ARMAX model. We assume that u_t is duly stationary, in other words the average independent of the time and equal to zero and the autocovariance of order s are only affected by the lapse of time between the periods and do not depend on the time.

In simple terms, we assume that there is a dynamic that can be estimated, and that consists of two parts: 1) a deterministic trend that may be approximated by a mathematical equation; and 2) a stochastic trend whose trajectory is influenced by the past values of the mortality rate and present and past innovations.

The first part of this dynamic (the deterministic trend) is considered in the ARMAX model as an exogenous explanatory variable (X), while the second "stochastic trend" is modeled with an autoregressive moving-average model.

The above two paragraphs can be shown in algebraic terms.

Solving Equation 11 and carrying out basic algebraic manipulations we obtain the variation of the mortality rate (Δy_t):

$$\begin{aligned}
 y_t &= (\phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p}) + (1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_p B^p) u_t + \alpha + \beta t + \lambda t^2 + \delta t^3 \\
 y_{t-1} &= (\phi_1 y_{t-2} + \phi_2 y_{t-3} + \dots + \phi_p y_{t-p-1}) + (1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_p B^p) u_{t-1} + \alpha + \beta(t-1) + \lambda(t-1)^2 + \delta(t-1)^3 \\
 \Delta y_{t-1} &= \underbrace{(\phi_1 \Delta y_{t-1} + \phi_2 \Delta y_{t-2} + \dots + \phi_p \Delta y_{t-p})}_{\text{Variación tasa de mortalidad}} + \underbrace{(\Delta u_t + \theta_1 \Delta u_{t-1} + \theta_2 \Delta u_{t-2} + \dots + \theta_p \Delta u_{t-p-1})}_{\text{Tendencia estocástica}} + \underbrace{(\beta + \lambda(2t-1) + \delta(3t^2 - 3t + 1))}_{\text{Tendencia determinista}}
 \end{aligned}$$

We do not impose a specific functional formula to estimate the deterministic trend; rather, we select a formula that adapts best to the data (we test within all the linear, quadratic and cubic equations). This selection is carried out using the Schwarz information criterion, which is consistent, i.e. for large samples it will tend to select the correct model if the assumptions are correct.

In algebraic terms the Schwarz criterion is:

Equation 14

$$SBIC = \ln(\delta^2) + (k/T) \ln T$$

However, for the purposes of the calculation we will use an equivalent formula.

Equation 15

$$SBIC = \ln\left(\sum_{t=1}^T \hat{u}_t\right) + \frac{k}{T} \ln T$$

Equation 15 is obtained using the fact that $E(u_t) = 0$, which implies $\hat{\sigma}^2 = \frac{\sum_{t=1}^T \hat{u}_t}{T}$; in

addition, as for all the models analyzed T is a constant, the only variation being k , minimizing equation 15 is equivalent to minimizing equation 14.

4.1. Non-parametric tests. Sample equivalence test. The Chilean case

As we commented above, when a determined country does not have reliable information available on mortality rates, it can adopt those of another country as its own, assuming that the two populations are similar. The differences in life expectancy may be adjusted by advancing/delaying the years with which the current mortality of the country in question is compared.

This methodology would not be appropriate for the case of Chile, as the country already has its own mortality tables (RV2004 and RV2009) that have been prepared following a major technical effort. However, RV04 and RV09, which are updated every five years, do not have a sufficient statistical history to project the relevant improvement factors in the long term.

The goal is therefore to project the Chilean mortality tables to measure the longevity risk that may occur in the future. To do so, we will compare the current mortality tables of Chile with those countries that have tables of sufficient quality and regularity of information so that by

projecting the tables of the country in question, we can obtain ones for Chile, under certain criteria of sample equivalence.

Forfar et al. (1988) have proposed a classic method of comparing mortality tables associated with different experiences. We can use this to establish the criterion that would allow us to state that two mortality tables of two different countries are statistically equivalent. **The authors propose non-parametric tests that compare the level of mortality (signs test), the form (runs test) and the existence of a similar pattern between both tables (χ^2 test).** The application of these tests will allow us to check what country presents mortality tables that are statistically comparable to the Chilean, N years ahead/behind.

We also apply an additional condition: that the equivalence of the tables must be stable and prolonged over time, so that this equivalence is not limited to a casual episode in time.

Based on the experiences of mortality in Chile (TCH) and in the country used for comparison (TPC), for which there are survivor series R_x^{TCH} and R_x^{TPC} to age x, where the total number

of deaths at age x is $A_x = A_x^{TCH} + A_x^{TPC}$, the probability of death would therefore be

$$q_x^{TCH} = \frac{A_x^{TCH}}{R_x^{TCH}} \text{ and } q_x^{TPC} = \frac{A_x^{TPC}}{R_x^{TPC}} \text{ respectively.}$$

For the purpose of applying non-parametric tests, we will begin by defining the null hypothesis and the alternative hypothesis. The null hypothesis is that there is no difference between the mortality of the group (TCH) and the mortality of the comparative country (TCP) (in other words, any difference between the mortality of the two populations is due to the sample or chance). The alternative hypothesis is that there is a difference between the two sets of data. Thus if it is rejected, it means that the data do not come from the same population. In this work, we have chosen a determined level of significance of 0.05, with one or two tails, as indicated.

The specific statistical form of test is selected according to each test. Most of the tests used are based on the hypothesis that the number of deaths at each age x A_x , can approximate a normal distribution, for $A_x \geq 5$, which may not occur for extreme ages. If this does not occur, the ages must be grouped together until the hypothesis is verified. That is why the data have been grouped together for ages $X \geq 84$.

Once the test statistic has been determined, we formulate the decision rule. This involves determining a number that separates the region where H_0 is not rejected from the region of rejection. This number is called the critical value and is determined by using the probability distribution associated with the test statistic, as well as the level of significance.

The decision rule is that it is not rejected if the calculated value of the test statistic is lower than the critical value. Essentially, this rule indicates that if there are major differences between the two mortality experiences, the null hypothesis H_0 should be rejected. Otherwise, the hypothesis is not rejected.

4.1.1. Signs test

In this test, the difference between the probability of death at age x, $q_x^{TCH} - q_x^{TPC}$ is calculated for each of the populations **TCH and TPC**.

The null hypothesis H_0 is that there is no difference between the mortality of group **TCH** (the mortality of the Chilean population) and the mortality of the group in the comparative country

(TPC). In other words, any difference in terms of the level of mortality of the comparison populations is due to the sampling.

The alternative hypothesis is that there is a difference between the two sets of data. If H_0 is **rejected**, means that the data do not come from the same population.

For the purpose of determining whether to reject H_0 , a level of significance is chosen. In our case a level of significance of 0.05 with two tails is selected.

The number of positive signs (NP) is chosen as the test statistic. On the hypothesis that it is true, there will be an equal probability (p=50%) of the differences being positive or negative. NP therefore follows a binomial distribution of N parameters, where in our case N is the number of age classes and **p=50%**:

$$P(NP = r) = \frac{N!}{r!(N-r)!} \frac{1}{2^N}, \quad r = 0, 1, 2 \dots N$$

$$E[NP] = \frac{N}{2} \quad y \quad V[NP] = \frac{N}{4}$$

For the purposes of determining whether to reject H_0 , a level of significance of **5% has been chosen, and H_0** is rejected if $P(NP = r) \leq 0,025$ or if $P(NP = r) \geq 0,025$.. In other words, the hypothesis that the mortality of each population is similar for very low values or very high for positive signs.

4.1.2. Runs test

In this test, the null hypothesis H_0 establishes that there is no difference in the form of the mortality table for the group TCH and the mortality of group TCP (in other words, any difference between the mortality of the two populations is due to the sampling). The alternative hypothesis H_1 is that there is a difference between the two sets of data. Thus if it is rejected, it means that the data do not come from the same population.

This test also calculates the difference between the probabilities of death at age x $q_x^{TCH} - q_x^{TPC}$, for each of the populations TCH and TPC and it is verified whether the signs of the difference are positive or negative.

Let n_1 be the number of positive signs and n_2 the number of negative signs, with $n_1 + n_2 = N$. The test statistic chosen is the number of groups with one or more consecutive deviations of the same sign (NR), which can be expressed as:

If $r=2k$ (if it is even):

$$P(NR = r) = \frac{2(n_1 - 1)!}{(k - 1)! (n_1 - k)!} \frac{(n_2 - 1)!}{(k - 1)! (n_2 - k)!} \frac{n_1! n_2!}{N!}$$

If $r=2k+1$ (if it is odd):

$$P(NR = r) = \frac{(n_1 - 1)!}{(k - 1)! (n_1 - K)!} \frac{(n_2 - 1)!}{k! (n_2 - 1 - K)!} \frac{n_1! n_2!}{N!} + \frac{(n_1 - 1)!}{k! (n_1 - K - 1)!} \frac{(n_2 - 1)!}{(k - 1)! (n_2 - K)!} \frac{n_1! n_2!}{N!}$$

The average number of runs is determined by:

$$E[NR] = \frac{2n_1 n_2}{N} + 1$$

And its variance is:

$$Var[NR] = \frac{2n_1 n_2 (2n_1 n_2 - N)}{N^2 (N - 1)}$$

For the purposes of determining whether to reject H_0 , a level of significance of 5% has been chosen and H_0 is rejected if $P(NP = r) \leq 0,025$ or if $P(NP = r) \geq 0,025$.

4.1.2 The chi2 test χ^2

In this test, the null hypothesis H_0 establishes that there is no difference between the mortality of the group TCH and the mortality of group TPC (in other words, any difference between the mortality of the two populations is due to the sampling). The alternative hypothesis H_1 is that there is a difference between the two sets of data. If H_0 is rejected, it means that the data do not come from the same population.

The following ratios are calculated for the purpose of applying the chi-squared test:

$$Z_X^{TCH} = \frac{A_X^{TCH} - R_X^{TCH} q_x}{\sqrt{R_X^{TCH} p_x q_x}} \quad \text{and} \quad Z_X^{TPC} = \frac{A_X^{TPC} - R_X^{TPC} q_x}{\sqrt{R_X^{TPC} p_x q_x}}$$

The test statistic is given by the following expression:

$$\chi^2 = \sum [(Z_X^{TCH})^2 + (Z_X^{TPC})^2]$$

If H_0 is true, the test statistic χ^2 follows a chi-squared distribution with N degrees of freedom. For the purposes of determining whether to reject H_0 , a level of significance of 5% has been chosen and therefore H_0 is rejected if $\chi^2 \geq 0.05$.

4.2 The data

The most extensive information available on mortality tables is included in the Human Mortality Database (HMD). The HMD5 represents a joint effort between the Department of Demography at the University of California, Berkeley and the Max Plank Institute for Demographic Research to prepare and compile detailed data on population and mortality and made available for the whole scientific community. Currently there is information for 37 countries, including Chile.

5 (<http://www.mortality.org/>)

Wilmoth et al. (2007) offer a broad and detailed description of the methodology used in the database.

According to Canudas-Romo (2008) the historical mortality series for Chile included in the HMD covers the period from 1992 to 2005. The author states that the main reasons for restricting the HMD series to the period since 1992 is that only in the last two censuses of 1992 and 2002 is the level of incorrect age information under 3%, and at least 90% of the deaths during this period were certified by a doctor.

The results

Comparing the Chilean tables from 1992 to 2005 with those of 22 countries⁶ in the database on which there is sufficient historical information to carry out econometric projections, we have obtained results that allow us to make a long-term projection for the case of Chile. Table 1 shows the countries in which the hypothesis of equivalence for the three types of tests proposed (signs, runs and chi) are not rejected. However, these countries would fail an additional condition that we have imposed on ourselves: that the equivalence relation in the tables should be stable over time and that it should cover a sufficient number of years. The only countries that comply with this condition would be **New Zealand** in the case of men, and **Austria** in the case of women. The dynamic generational tables for Chile are equivalent to Austria -4 years for men and to New Zealand -6 years for women.

Table 1
Non-parametric tests to identify similarities at the age of 65 years old

Men /Austria													
	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Signs	1	1	1	0	0	1	1	1	1	1	1	1	1
Runs test	1	0	1	1	1	0	1	1	1	1	1	1	1
Chi	1	1	1	1	1	1	1	1	1	1	1	1	1
Chile	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004

Women / New Zeland														
	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Signs	1	1	1	1	1	1	1	0	1	1	1	1	1	1
Runstest	1	1	1	1	1	1	0	1	1	1	0	1	1	1
Chi	1	1	1	1	1	0	1	1	1	1	1	1	1	1
Chile	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007

1=The null hypothesis of sample equivalence is accepted.

0=The null hypothesis of sample equivalence is rejected.

Source: BBVA Research

⁶ The countries for which we have carried out the sample equivalent tests are: Spain, Denmark, Slovenia, Portugal, Finland, Ireland, the United Kingdom, Germany, Belgium, New Zealand, Holland, Austria, Norway, Canada, Israel, Sweden, Australian France, Iceland, Italy, Switzerland and Japan.

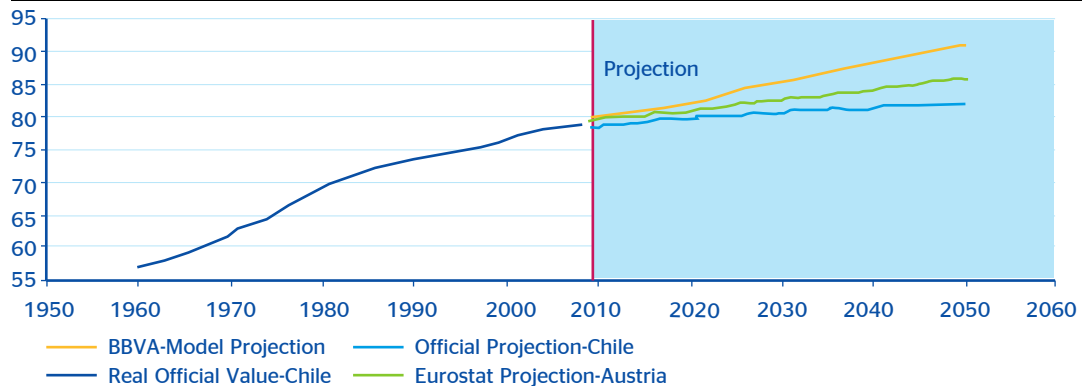
The long-term relationship tested using the available data shows that for the period 1992-2004 for men and 1994-2007 in the case of women, in most cases (a green "1" in Table 1 above), the three tests are significant and do not reject the sample equivalence hypothesis. Only in a few cases (a white "0") is the equivalence hypothesis rejected in one of the tests. A test of the robustness is that the most statistically potent test (chi) only fails in the case of women in 1993.

The generational tables for Austria and New Zealand have been available since 1948, so we can make the appropriate econometric analyses to project the tables of these countries in the long-term t, and thus those of Chile, with their corresponding gender lags.

The projections based on the ARMAX model by specific ages offer a life expectancy of 90.91 years in 2050, while the Chilean National Institute of Statistics (INE) projects a life expectancy at birth of 82.14 years (see Chart 2).

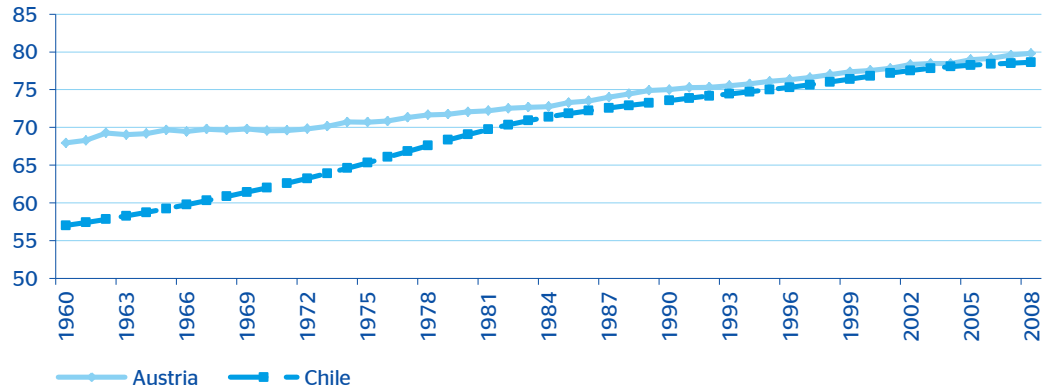
If we take the official estimates of the INE (82.14) and compare it with the estimates of Eurostat for the case of Austria (86.5) the life expectancies in Chile and Austria can be seen to diverge by around 4 years, unlike what one would expect given the historical trend of these two countries. In fact, one can see in Chart 3 that starting in 1985, the difference between the projections of life expectancy in Austria and Chile have remained relatively constant at around 1.6 years on average to 2008.

Chart 2
Life expectancy projections at the age of 65



Source: BBVA Research

Chart 3
Convergence of the life expectancies



Source: www.mortality.org

To sum up, the INE projections for Chile show a divergence in the life expectancy of Chile and Austria in 2050, which is contrary to the empirical evidence observed for the last 50 years, both in the projection carried out by Eurostat and in that carried out for this paper. These discrepancies and the extent of the deviation may generate uncertainty: for the insurance industry when calculating life annuities; and in terms of a possible insufficiency of the contribution rates needed to achieve an adequate replacement rate for future pensioners due to an underestimated life expectancy.

5. The economic impact of the use of inadequate mortality tables

A life annuity is an insurance by which, in exchange for a premium, an insurance company guarantees the payment of a regular income until the rights expire. This income to be paid by the insurance company includes payment of interest at what is called a technical interest rate. When the time for retirement comes, a pension saver transfers the accumulated capital in his or her individual capitalization account to an insurance company and the company assumes the financial and longevity risks associated with the product.

The fundamental variables of the life annuity business are, first, the financial variables, interest rates; and second, changes in the mortality rates and, in particular, longevity risk. The insurance company guarantees an interest rate throughout the lifetime of the insurance. From the financial point of view, the insurance company therefore has to be able to invest the funds in financial assets that can guarantee its commitment to the insured party.

The time horizon of the operation is also uncertain, depending basically on the life expectancy of the insured party. If the life expectancy of a population increases with respect to the life expectancy considered at the time the rates for the product were set, this would involve a loss for the company, as it would have to pay more monthly payments than it had initially calculated. Therefore, the life annuity business must take changes in the underlying variables into account if it is to be viable, as an unfavorable change in any of them will result in the amount charged being insufficient a particular moment of time.

The product's fundamental variables have to be determined when setting the rates for life annuities. This involves defining the biometric bases; in other words, how the mortality of the population will progress over time and what interest rate will be guaranteed. Below we analyze in detail how a life annuity is calculated, and how the changes in the underlying variables, above all changes in mortality, affect this.

5.1 Setting pension levels for life annuities

Let q_x be the probability of dying at age x ; the probability of dying in the following years is $q_{x+1}, q_{x+2}, \dots, q_{x+h}$. These probabilities are published in tables derived from mortality studies covering a specific population.

Let p_{x+h} be the probability of being alive at time $x + h$, conditioned on the individual being alive at time x . This probability is defined as follows:

$$P_{x+h} = \prod_{i=0}^h (1 - q_{x+i})$$

Let r be the technical rate, i.e. the interest rate guaranteed in the life annuity. The discount factor for time h is defined as:

$$fd_h = \frac{1}{(1+r)^h}$$

The technical capital required, CTN, to buy a life annuity is defined as the current value discounted at the technical rate of the pension weighted by the probability of being alive. In general, for a person of age x this is expressed as follows:

$$CTN_x = \sum_{i=0}^N pension * p_{x+i} * fd_i$$

where N is the maximum age included in certain biometric bases.

As can be seen in the above calculations, once the biometric bases, the technical rate and the amount of the pension have been set, the technical capital needed to buy a life annuity is fully determined. Equally, once the biometric bases, the technical rate and a certain amount of capital have been set, the pension payable can be deduced, as shown below:

$$pensión = \frac{CTN_x}{\sum_{i=0}^N P_{x+i} * fd_i}$$

where N is the maximum age included in certain biometric bases.

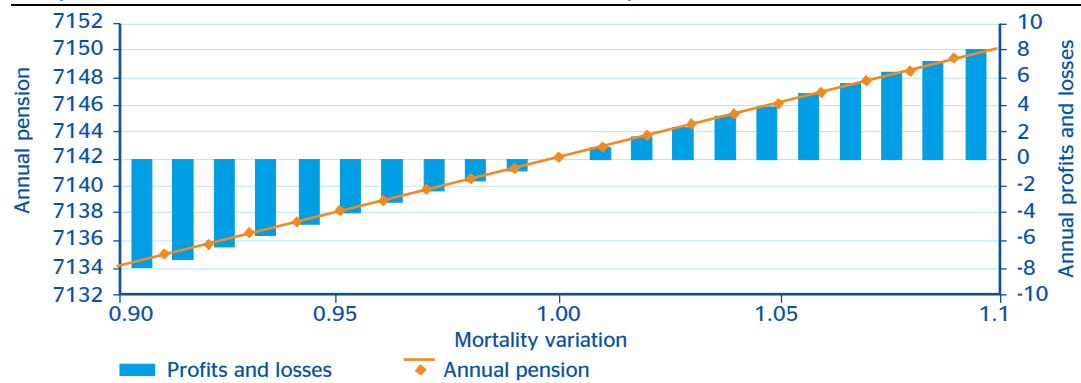
5.2 Sensitivity analysis

Once the pension or technical capital needed to buy a life annuity has been determined, the amount that the insurance company will pay to the insured party is established. In other words, the process of setting pension rates includes the establishment of the cost of the operation under certain biometric and market hypotheses.

The term of the operation is uncertain and will depend on the biometrics. In addition, the operation of life annuities is generally very long-term, which involves a great deal of uncertainty associated with market variables. During the lifetime of the operation, the underlying variables change, and as a result the rate set initially may not be sufficient to cover the associated costs of the operation.

The following exercise analyzes how changes in the biometrics affect the pension. The baseline scenario is set as the pension that a man reaching retirement age at 65 will receive, taking into account the biometric bases of the mortality table RV2009, a technical rate of 3.5% and a capital saved of 100,000 monetary units. The table is then modified, multiplying the probabilities of mortality by a percentage and revaluing the pension while maintaining the other variables constant. Chart 4 shows the results obtained as follows:

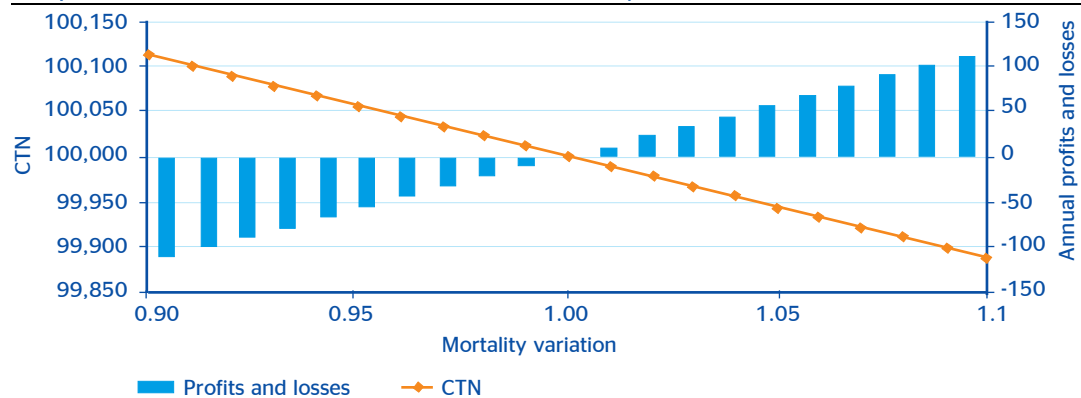
Chart 4
Losses and gains in the industry and pension benefits in the event of variations in mortality (in USD)



Source: BBVA Research

It can be seen that if we consider table RV2009 as the baseline scenario of mortality, the reduction in mortality (left) involves an insufficient rate and increased mortality (right) involves a sufficient rate. Chart 5 shows how the technical capital required varies as the mortality varies for a pre-set pension.

Chart 5
Technical Capital in the industry and pension benefits in the event of variations in mortality (in USD)



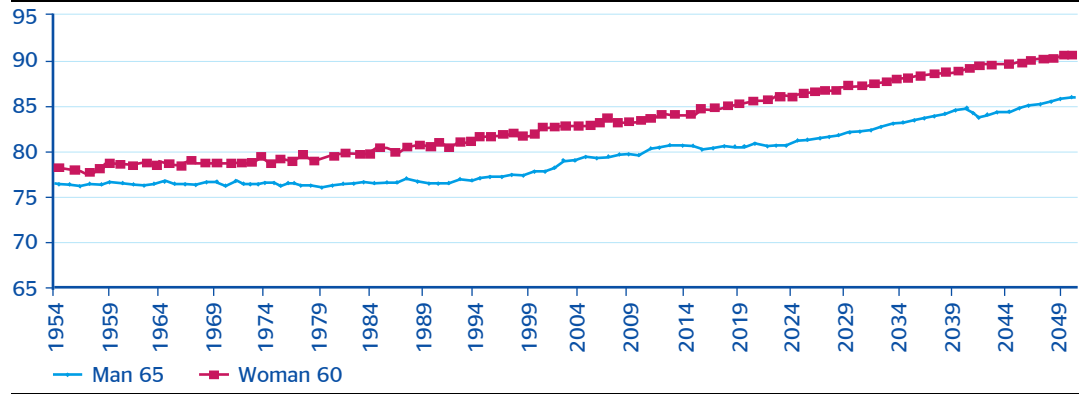
Source: BBVA Research

The above chart shows that maintaining pensions constant and being able to guarantee a certain standard of living as mortality changes and life expectancy increases means the accumulated capital must necessarily increase. Once the rates and sensitivities associated with this process have been clarified, the object of this study is now to quantify the pension rate deficit for the Chilean pension system over time.

The retirement age in Chile is 65 for men and 60 for women, so these model points were used for all the calculations. For the biometric bases, we used mortality tables projected from 1954 to 2050 to determine the pension rate over time. The interest rate used is 3.5%. Chart

6 shows how life expectancy will change for a man aged 65 and a woman aged 60 over time using the projections calculated in this work.

Chart 6
Life expectancy

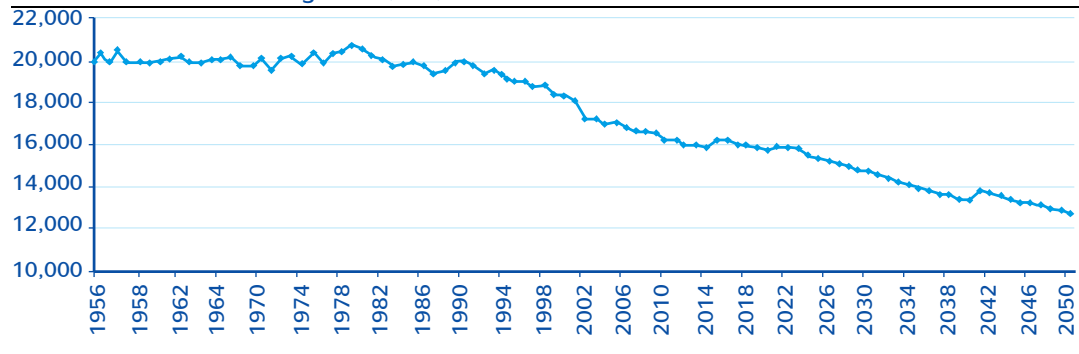


Source: BBVA Research

It can be seen that although life expectancy for a man aged 65 in 1954 was 76.5, in 2050 it will have risen to 86. Similarly, a woman aged 60 in 1954 had a life expectancy of 78, and in 2050 it will be 90.5.

The increased life expectancy, or the equivalent reduction in mortality, means that for a specific amount of capital, the monthly income to be received by a person who retires at a certain point in time will be greater than the income to be received by another person who retires at a later time. Given the above, Chart 7 shows the differences in the pensions to be received by a man aged 65 and a woman aged 60 who retire between 1954 and 2050.

Chart 7
Annual income for a man aged 65 (USD)

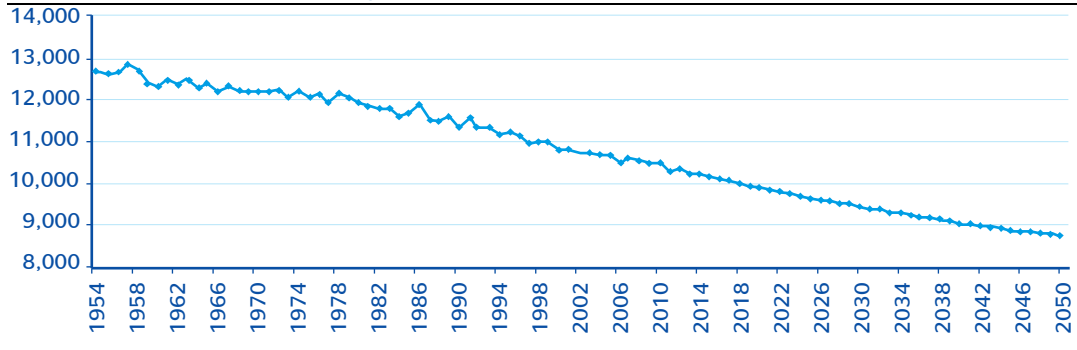


Source: BBVA Research

A man aged 65 who retired in 1954 with 190,000 dollars saved would receive a pension of an annual 20,000 dollars. Currently, the same man aged 65 would receive a pension of 16,000 dollars, and this amount would fall to 12,700 dollars in 2050. In other words, the fact that mortality falls over time involves a reduction of 20% in pensions over the next 40 years.

Analogously to the above case, a woman aged 60 who retired in 1954 with savings of 167,000 dollars would receive an annual income of 12,600 dollars. Currently this pension would fall to 10,300 dollars and in 2050 it will be 8,700 dollars. This represents a reduction of 15% over the next forty years (see Chart 8).

Chart 8
The annual income of a woman aged 60 years (in USD)

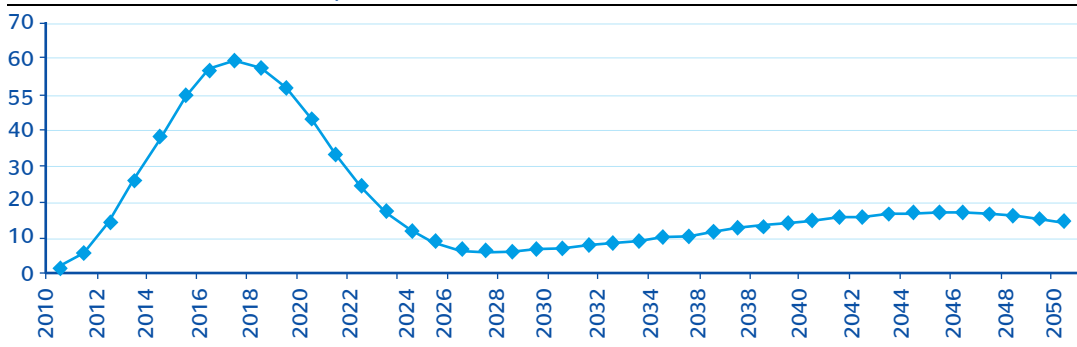


Source: BBVA Research

In the previous section we saw that there are discrepancies in the predictions made by different institutions; in the specific case of Austria and Chile, it is notable that according to some institutions, the convergence of life expectancies in the two countries will be reversed over the next 40 years. This is perhaps due to different calculation methodologies. The question to ask, therefore, is Who is right? The risk of an incorrect answer to this question is very relevant for the pension industry.

A simulation exercise shows us that a 1% deviation in the mortality tables used for calculating life annuities would result in losses for the industry that could amount to 60 million dollars in 2017 (see Chart 9). **These losses would be mainly due to two factors: the effect of greater longevity of pension savers and the effect of the use of inadequate pension rates.**

Chart 9
Systemic risk simulation in the use of mortality tables with a 1% error in the mortality tables of Chile (in million dollars)

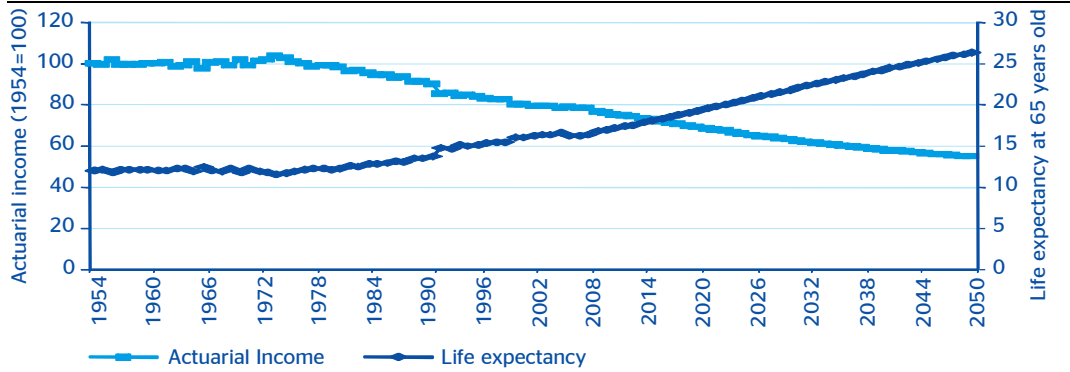


Source: BBVA Research

In addition, increased life expectancy could mean that pension savers in many Latin American countries may not be making sufficient contributions to their pension funds. An error in estimating this life expectancy could imply that pension savers would have to distribute their accumulated balances in a life annuity over a longer period of time, so their available income would reduce their replacement rate and thus their standard of living.

If extra contributions were not made, future generations could see their retirement pensions reduced by nearly 50% due to increased life expectancy (see Chart 10).

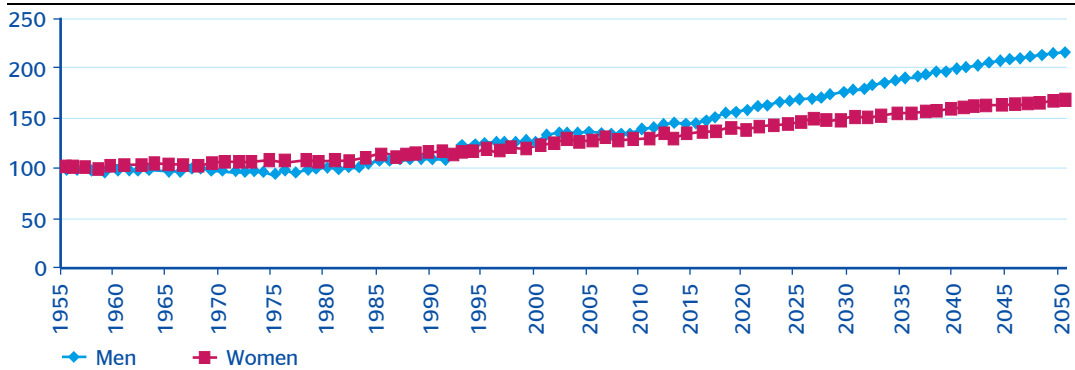
Chart 10
Evolution in life expectancy and the pensions of the men



Source: BBVA Research

In another simulation exercise, we have calculated what the contribution rate would have to be in Chile to maintain the current replacement rate (see Chart 11). The contribution rate would have to increase by an average of 8 percentage points in the case of men and 4 percentage points for women on current levels. The difference by gender can be explained by the greater relative increase in men's life expectancy compared with women (a convergence can be observed between the genders), and because men have higher salaries and thus have to accumulate a higher capital balance to maintain the replacement rate.

Chart 11
Simulation of required contribution rates to maintain current rate of substitution



Source: BBVA Research

All these results point to the need to calculate good long-term dynamic mortality tables to mitigate the longevity risks for both the industry and pension savers. If Chile, which currently has the best tables in the region, is facing these risks, the need for other countries is even more pressing.

6. Conclusions

Increased life expectancy at birth poses major challenges for society, given the important transformations that will influence the ways in which supply and demand will interact in the future. Industries that are most closely linked to longevity risks, such as insurance and pensions, have been incorporating major changes in their business processes to allow them to interact as well as possible. One of the key elements of these markets is the appropriate projections of mortality rates, based on a set of variables that anticipate the trends in future life expectancy and thus to ensure a correct estimate of the risks. The "art" of constructing

mortality tables has been developed over time. It requires a detailed and ongoing analysis of all those aspects that affect the possibility of living longer, as well as anticipating future trends based on current events. A number of developed countries have developed a working history based on a long tradition of collecting historical mortality data, which has allowed them to narrow the gaps with the real situation. However, this progress has a very limited history in Latin America. As a result, the region is faced with major challenges in the future to incorporate technical progress and the information needed for the construction of improved tables.

This work has used the case of Chile to reveal how the mortality tables in Latin America may be lagging behind reality, and the consequent economic effects on the insurance and pension industry. The calculations have only been made for Chile, as it is the country with most available information for applying the calculation methodology used in the project.

The structure of the mortality tables in Chile has been compared with that of the 22 countries where there is sufficient historical information to carry out econometric projections. This analysis uses non-parametric tests that compare the level of mortality (signs test), the form (runs test) and the existence of a similar pattern between both tables (χ^2 test).

The countries that are statistically most similar to the Chilean case are New Zealand in the case of men and Austria in the case of women. The dynamic generational tables for Chile are equivalent to Austria -4 years for men and New Zealand -6 years for women.

The long-term relationship tested using the available data shows that for the period 1992-2004 for men and 1994-2007 in the case of women, in most cases the three tests are significant and do not reject the sample equivalence hypothesis. Only in a few cases is the equivalence hypothesis rejected in one of the tests. A test of robustness is that the statistically most potent test (chi) fails only in the case of women for the year 1993. The generational tables for Austria and New Zealand are available from 1948, which has enabled the appropriate econometric analyses to project the tables of these countries in the long-term; because of the equivalence this meant the same could be done for Chile, with the corresponding lag by gender.

For the projection exercises, the mortality rates were calculated using ARMA(p, q) models for each of the 101 ages and for each gender, with a total of 202 ARMA(p,q) regressions. The results offer an average life expectancy (men and women) of 90.91 years in 2050, while the INE in Chile projects a life expectancy at birth of 82.14 years. If we take the official estimates of the INE (82.14) and compare them with the estimates of EuroPop for the case of Austria (86.5) it is noticeable that the life expectancy of Chile and Austria diverge by around 4 years, unlike what one would expect given the historical trend of these two countries. In fact, one can see that starting in 1985, the difference between the projected life expectancies for Austria and Chile have remained relatively constant at around 1.6 years on average until 2008. Thus the INE projections for Chile would show a divergence between the life expectancy of Chile and Austria in 2050 that is contrary to the observed empirical evidence over the last 50 years, in both the Eurostat projection and in this paper. These discrepancies and the extent of the deviation may generate uncertainty: for the insurance industry when calculating life annuities; and in terms of a possible insufficiency of the contribution rates needed to achieve an adequate replacement rate for future pensioners due to an underestimated life expectancy.

A simulation exercise shows us that a deviation of 1% in the mortality tables used for calculating life annuities would result in losses for the industry that could amount to 60 million dollars in 2017. These losses would mainly be due to two factors: the effect of greater longevity of the pension savers and the use of inadequate pension rates. In addition, the increased life expectancy could imply that pension savers in many Latin American countries are not making sufficient contributions to their pension funds. An error in estimating this life expectancy would mean that pension savers would have to distribute their accumulated balances in a life annuity over a longer period of time, so their available income would reduce their replacement rate and thus their standard of living. If extra contributions were not made,

future generations could see their retirement pensions reduced by nearly 50% due to increased life expectancy.

In another simulation exercise, we also calculated what the contribution rate would have to be in Chile to maintain the current replacement rate. The contribution rate would have to increase by an average of 8 percentage points in the case of men and 4 percentage points for women on current levels. The difference by gender can be explained by the greater relative increase in men's life expectancy compared with women (a convergence can be observed between the genders), and because men have higher salaries and thus have to accumulate a higher capital balance to maintain the replacement rate.

Bibliography

Alonso Meseguer, Javier; Sosvilla Rivero, Simón (2007), "Proyecciones de tablas de mortalidad dinámicas de España y sus comunidades autónomas", FEDEA

Camarda (2008) "Smoothing methods for the analysis of mortality development" Universidad Carlos III de Madrid. Department of Statistics. PhD Thesis.

Canudas-Romo, V. (2008), "The modal age at death and the shifting mortality hypothesis". *Demographic Research*, 19(30):1179-1204.

Gompertz B. (1825) "On the nature of the function expressive of the law of human mortality and on a new mode of determining life contingencies. *Philosophical Transactions of the Royal Society of London*, A 115, 513-585.

Heligman, L. and Pollard, J. H. (1980). "The Age Pattern of Mortality," *Journal of the Institute of Actuaries* 107:49-75.

Himes C.L. et al. (1994). "A Relational Model of Mortality at Older Ages in Low Mortality Countries," *Population Index* 48:269-91.

Lee, R.D. and Carter, L. (1992) "Modeling and Forecasting the Time Series of U.S. Mortality," *Journal of the American Statistical Association* 87:659-71.

Makeham W. M. (1860) "On the Law of Mortality and the Construction of Annuity Tables". *J. Inst. Actuaries and Assur. Mag.* 8: 301-310.

Perks W. (1932) "On some experiments on the graduation of mortality statistics", *Journal of the Institute of Actuaries*, 63, 12-40.

Pitacco, E. et al, 2009. "Modeling longevity dynamics for pensions and annuity business". Oxford University Press, Oxford.

Prentice, Thomson (2006) "Health, history and hard choices: Funding dilemmas in a fast-changing world" *Health and Philanthropy: Leveraging change*. University of Indiana. World Health Organization. August 2006

Thatcher A. R. (1999) "The long-term pattern of adult mortality and the highest attained age". *Journal of the Royal Statistical Society (A)* 162(1): 5-43.

Siler, W. (1983) "Parameters of mortality in human populations with widely varying life spans". *Statistics in Medicine*, 2: 373-380.

Superintendencia de Banca, Seguros y AFP, República del Perú (2006), "Memoria Anual 2006"

Superintendencia de Administradoras de Fondos de Pensiones, Chile, (1990), "Circular N° 656 sobre Normas para el Otorgamiento de Pensiones y Otros Beneficios"

Superintendencia de Administradoras de Fondos de Pensiones, Chile, (2004), "Circular N° 1314 Fija Tabla de Mortalidad RV-2004, Hombres y Mujeres, para pensionados por vejez"

Superintendencia de Administradoras de Fondos de Pensiones, Chile, y el Gobierno de Chile (2004), "Comunicado de Prensa Conjunto - Norma sobre nuevas tablas de mortalidad"

Superintendencia Bancaria de Colombia (2000), "Circular externa 071/2000 sobre Estadísticas de mortalidad de activos, de rentistas, de inválidos y de invalidez"

Superintendencia Financiera de Colombia (2010), "Resolución N° 1555/2010 por la cual se actualizan las Tablas de Mortalidad de Rentistas Hombres y Mujeres"

Superintendencia de Banca y Seguros, Perú (1993), "Resolución N° 309/93 por la que se autoriza el uso de la tabla chilena de mortalidad RV- 85, B-85, MI-85"

Superintendencia de Banca y Seguros, Perú (2006), "Resolución N° 354/2006 por la que se aprueba el uso obligatorio de las tablas RV-2004 Modificada"

Superintendencia de Banca y Seguros, Perú (2010), "Resolución N° 17728/2010 por la que se aprueba el uso obligatorio de las tablas RV-2004 Modificada Ajustada, B-85 Ajustada"

Wilmoth J.R. et al (2007) "Methods Protocol for the Human Mortality Database".

Vegas Asensio, Jesús María (2000) "El riesgo de longevidad en los planes de pensiones", Anales del Instituto de Actuarios Españoles, N°6, págs. 119-157

Working Papers

2013

13/01 **Hugo Perea, David Tuesta y Alfonso Ugarte:** Lineamientos para impulsar el Crédito y el Ahorro. Perú.

13/02 **Ángel de la Fuente:** A mixed splicing procedure for economic time series.

13/03 **Ángel de la Fuente:** El sistema de financiación regional: la liquidación de 2010 y algunas reflexiones sobre la reciente reforma.

13/04 **Santiago Fernández de Lis, Adriana Haring, Gloria Sorensen, David Tuesta, Alfonso Ugarte:** Lineamientos para impulsar el proceso de profundización bancaria en Uruguay.

13/05 **Matt Ferchen, Alicia Garcia-Herrero and Mario Nigrinis:** Evaluating Latin America's Commodity Dependence on China.

13/06 **K.C. Fung, Alicia Garcia-Herrero, Mario Nigrinis Ospina:** Latin American Commodity Export Concentration: Is There a China Effect?.

13/07 **Hugo Perea, David Tuesta and Alfonso Ugarte:** Credit and Savings in Peru.

13/08 **Santiago Fernández de Lis, Adriana Haring, Gloria Sorensen, David Tuesta, Alfonso Ugarte:** Banking penetration in Uruguay.

13/09 **Javier Alonso, María Lamuedra y David Tuesta:** Potencialidad del desarrollo de hipotecas inversas: el caso de Chile.

13/10 **Ángel de la Fuente:** La evolución de la financiación de las comunidades autónomas de régimen común, 2002-2010.

13/11 **Javier Alonso, María Lamuedra y David Tuesta:** Potentiality of reverse mortgages to supplement pension: the case of Chile.

13/12 **Javier Alonso y David Tuesta, Diego Torres, Begoña Villamide:** Proyecciones de tablas generacionales dinámicas y riesgo de longevidad en Chile.

13/13 **Alicia Garcia Herrero and Fielding Chen:** Euro-area banks' cross-border lending in the wake of the sovereign crisis.

13/14 **Maximo Camacho, Marcos Dal Bianco, Jaime Martínez-Martín:** Short-Run Forecasting of Argentine GDP Growth.

13/15 **Javier Alonso y David Tuesta, Diego Torres, Begoña Villamide:** Projections of dynamic generational tables and longevity risk in Chile.

2012

12/01 **Marcos Dal Bianco, Máximo Camacho and Gabriel Pérez-Quiros:** Short-run forecasting of the euro-dollar exchange rate with economic fundamentals.

12/02 **Guoying Deng, Zhigang Li and Guangliang Ye:** Mortgage Rate and the Choice of Mortgage Length: Quasi-experimental Evidence from Chinese Transaction-level Data.

12/03 **George Chouliarakis and Mónica Correa-López:** A Fair Wage Model of Unemployment with Inertia in Fairness Perceptions.

12/04 **Nathalie Aminian, K.C. Fung, Alicia García-Herrero, Francis NG:** Trade in services: East Asian and Latin American Experiences.

12/05 **Javier Alonso, Miguel Angel Caballero, Li Hui, María Claudia Llanes, David Tuesta, Yuwei Hu and Yun Cao:** Potential outcomes of private pension developments in China (Chinese Version).

12/06 **Alicia Garcia-Herrero, Yingyi Tsai and Xia Le:** RMB Internationalization: What is in for Taiwan?

- 12/07 **K.C. Fung, Alicia Garcia-Herrero, Mario Nigrinis Ospina:** Latin American Commodity Export Concentration: Is There a China Effect?
- 12/08 **Matt Ferchen, Alicia Garcia-Herrero and Mario Nigrinis:** Evaluating Latin America's Commodity Dependence on China.
- 12/09 **Zhigang Li, Xiaohua Yu, Yinchu Zeng and Rainer Holst:** Estimating transport costs and trade barriers in China: Direct evidence from Chinese agricultural traders.
- 12/10 **Maximo Camacho and Jaime Martinez-Martin:** Forecasting US GDP from small-scale factor models in real time.
- 12/11 **J.E. Boscá, R. Doménech and J. Ferria:** Fiscal Devaluations in EMU.
- 12/12 **Ángel de la Fuente and Rafael Doménech:** The financial impact of Spanish pension reform: A quick estimate.
- 12/13 **Biliana Alexandrova-Kabadjova, Sara G. Castellanos Pascacio, Alma L. García-Almanza:** The Adoption Process of Payment Cards -An Agent- Based Approach .
- 12/14 **Biliana Alexandrova-Kabadjova, Sara G. Castellanos Pascacio, Alma L. García-Almanza:** El proceso de adopción de tarjetas de pago: un enfoque basado en agentes.
- 12/15 **Sara G. Castellanos, F. Javier Morales y Mariana A. Torán:** Análisis del uso de servicios financieros por parte de las empresas en México: ¿Qué nos dice el Censo Económico 2009?
- 12/16 **Sara G. Castellanos, F. Javier Morales y Mariana A. Torán:** Analysis of the Use of Financial Services by Companies in Mexico: What does the 2009 Economic Census tell us?
- 12/17 **R. Doménech:** Las Perspectivas de la Economía Española en 2012:
- 12/18 **Chen Shiyuan, Zhou Yinggang:** Revelation of the bond market (Chinese version).
- 12/19 **Zhouying Gang, Chen Shiyuan:** On the development strategy of the government bond market in China (Chinese version).
- 12/20 **Angel de la Fuente and Rafael Doménech:** Educational Attainment in the OECD, 1960-2010.
- 12/21 **Ángel de la Fuente:** Series enlazadas de los principales agregados nacionales de la EPA, 1964-2009.
- 12/22 **Santiago Fernández de Lis and Alicia Garcia-Herrero:** Dynamic provisioning: a buffer rather than a countercyclical tool?.
- 12/23 **Ángel de la Fuente:** El nuevo sistema de financiación de las Comunidades Autónomas de régimen común: un análisis crítico y datos homogéneos para 2009 y 2010.
- 12/24 **Beatriz Irene Balmaseda Pérez y Lizbeth Necochea Hasfield:** Metodología de estimación del número de clientes del Sistema Bancario en México.
- 12/25 **Ángel de la Fuente:** Series enlazadas de empleo y VAB para España, 1955-2010.
- 12/26 **Oscar Arce, José Manuel Campa y Ángel Gavilán:** Macroeconomic Adjustment under Loose Financing Conditions in the Construction Sector.
- 12/27 **Ángel de la Fuente:** Algunas propuestas para la reforma del sistema de financiación de las comunidades autónomas de régimen común.
- 12/28 **Amparo Castelló-Climent, Rafael Doménech:** Human Capital and Income Inequality: Some Facts and Some Puzzles.
- 12/29 **Mónica Correa-López y Rafael Doménech:** La Internacionalización de las Empresas Españolas.
- 12/30 **Mónica Correa-López y Rafael Doménech:** The Internationalisation of Spanish Firms.

12/31 **Robert Holzmann, Richard Hinz and David Tuesta:** Early Lessons from Country Experience with Matching Contribution Schemes for Pensions.

12/32 **Luis Carranza, Ángel Melguizo and David Tuesta:** Matching Contributions for Pensions in Colombia, Mexico, and Peru: Experiences and Prospects.

12/33 **Robert Holzmann, Richard Hinz y David Tuesta:** Primeras lecciones de la experiencia de países con sistemas de pensiones basados en cotizaciones compartidas.

12/34 **Luis Carranza, Ángel Melguizo y David Tuesta:** Aportaciones compartidas para pensiones en Colombia, México y Perú: Experiencias y perspectivas.

[Click here to access the list of Working Papers published between 2009 and 2011](#)

The analysis, opinions, and conclusions included in this document are the property of the author of the report and are not necessarily property of the BBVA Group.

BBVA Research's publications can be viewed on the following website: <http://www.bbvarsearch.com>

Contact Details

BBVA Research

Paseo Castellana, 81 - 7th floor
28046 Madrid (Spain)
Tel.: +34 91 374 60 00 and +34 91 537 70 00
Fax: +34 91 374 30 25
bbvarsearch@grupobbva.com
www.bbvarsearch.com